

# Results on Energy- and Spectral-Efficiency Tradeoff in Cellular Networks With Full-Duplex Enabled Base Stations

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**Abstract**—In this paper, we address the tradeoff between energy efficiency (EE) and spectral efficiency (SE) for cellular networks with full-duplex (FD) communications enabled base stations. To be backward compatible with legacy LTE systems, it is assumed that user devices still work in the conventional half-duplex (HD) mode. There usually exists residual self-interference (RSI) in FD communications after advanced interference suppression techniques are applied. In this paper, we consider two different RSI models: constant RSI model and linear RSI model. First, the necessary conditions for an FD transceiver to achieve better EE-SE tradeoff than an HD one are derived for both the RSI models. Then, for the constant RSI model, a closed-form EE-SE expression is obtained in the scenario of single pair of users. We further extend our result and prove that EE is a quasi-concave function of SE in the scenario of multiple user pairs. Accordingly, an optimal algorithm to achieve the maximum EE based on the Lagrange dual decomposition technique is developed. For the linear RSI model, the EE-SE relation is difficult to deal with and we develop a heuristic algorithm by decoupling the problem into two sub-problems: power control and resource allocation. Our analysis and algorithms are finally verified by comprehensive numerical results.

**Index Terms**—Full-duplex communications, energy-efficiency, spectral-efficiency, tradeoff.

## I. INTRODUCTION

**F**UTURE cellular networks are facing the challenging task of increasing the network capacity to support large

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amounts of mobile data communications. To meet such a challenge, full-duplex (FD) communications have been recently proposed, which enable a device to transmit and receive simultaneously on the same spectrum and can potentially double the wireless link capacity [2]. The main challenge of implementing FD communications is to deal with self-interference (SI), caused by the FD device itself, which affects the desired signal of the same device. With current SI cancellation techniques, such as propagation domain suppression [3], [4], analog cancellation [5], and digital cancellation [6], the SI can be cancelled to a sufficiently low level, to make the FD communications practically implementable in wireless systems.

There have been a number of studies investigating the spectral-efficiency (SE) enhancement of FD communications enabled cellular networks. In [7], a medium access control (MAC) protocol called asymmetrical duplex has been developed to efficiently support the coexistence of FD and conventional half-duplex (HD) communications. The performance of a hybrid full/half-duplex heterogeneous network has been analyzed in [8]. A joint subcarrier assignment and power allocation algorithm to maximize the sum-rate performance of a cellular network with FD enabled base stations (BSs) and mobile devices has been developed in [9]. Leveraging the opportunistic interference cancellation (OIC) technique to cancel the co-channel interference (CCI), a joint mode selection and resource allocation algorithm to maximize the overall system throughput has been developed in [10]. Moreover, a sub-optimal user pairing and time-slot allocation algorithm has been proposed for the time-division (TD) FD network in [11].

On the other hand, since power consumption of mobile devices is increasing rapidly whereas the battery capacity is still limited, energy-efficiency (EE) becomes a more and more important metric for future cellular networks. The EE design of wireless systems has been investigated since a decade ago. It is well known that the EE and SE cannot be simultaneously optimized in general, especially when the circuit power consumption is considered. Therefore, it is important to analyze the EE-SE relation or tradeoff to guide the design of spectral-efficient and energy-efficient communications.

The EE-SE tradeoff has been initially investigated in [12], which has proved that EE is a quasi-concave function on SE for downlink OFDMA networks. Later, the EE-SE tradeoff for downlink OFDMA networks has been further investigated by taking into account the user fairness in [13].

The EE-SE tradeoff has been also extended into the amplify-and-forward based relay network, which demonstrates that the EE-SE tradeoff curve is still quasi-concave in this scenario [14]. In [15] and [16], the EE-SE tradeoff in type-I ARQ system and cognitive radio has been studied, respectively. The EE-SE study in homogeneous cellular networks with random distributed BSs shows that, with respect to the outage constraint, the EE-SE tradeoff only occurs under a given network situation [17]. In [18], the EE-SE tradeoff has been investigated for video transmission over mobile Ad Hoc networks. In [19] and [20], an alternative multi-objective optimization approach has been applied to analyze the EE-SE tradeoff. Again, the quasi-concave EE-SE relation is also revealed. Leveraging the stochastic geometry method, a theoretical framework for analyzing the EE-SE tradeoff in multi-cell cellular networks to achieve tractable results has been developed in [21], recently.

However, the studies mentioned above mainly focus on the conventional HD network while the EE-SE tradeoff for the FD network has not been addressed yet. In the FD network, an uplink user and a downlink user are paired to transmit on the same channel simultaneously, causing two kinds of interference: SI at the BS affecting the uplink transmission and the CCI at the user side affecting the downlink transmission. These new kinds of interference render it difficult to analyze the EE-SE tradeoff for the FD network. In particular, it is rather challenging to analyze the non-convex EE maximization problem under a given SE as well as address the joint user pairing and resource allocation problem. Moreover, it is still an open question whether the EE-SE relation preserves quasi-concavity for the FD network.

To address the above issues, we investigate the EE-SE tradeoff in the FD network with two different residual self-interference (RSI) models: constant RSI model [9] and linear RSI model [22]. The main contributions of this work can be summarized as follows.

- We find the necessary conditions for FD communications to achieve better EE-SE tradeoff than the conventional HD communications for both constant and linear RSI models.
- For the constant RSI model, we derive a closed-form solution to the EE maximization problem and prove that the EE is a quasi-concave function of the SE in the scenario of single pair of users. Furthermore, we extend proving the quasi-concavity of the EE-SE relation into the scenario of multiple user pairs. According to this, a global optimal solution to achieve the maximum EE for a given SE region is developed.
- Different from the constant RSI model, the linear RSI model is a little bit difficult to deal with. We first solve the EE-SE tradeoff problem numerically in the scenario of single pair of users and then develop a heuristic power control and user pairing algorithm in the scenario of multiple user pairs.

The rest of the paper is organized as follows. In Section II, we describe the system model for the FD network and formulate the EE-SE tradeoff problem. In Section III, necessary

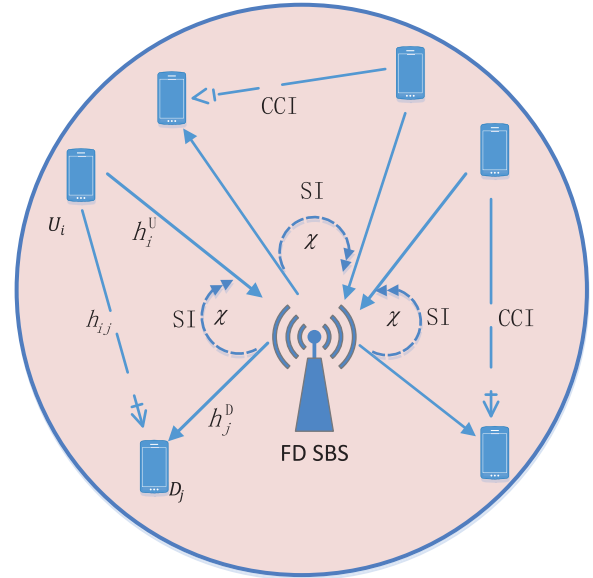


Fig. 1. System model of FD enabled cellular networks.

conditions are derived for FD communications to be better than HD communications in term of EE-SE tradeoff for the two RSI models. In Section IV, the EE-SE tradeoff problem for the constant RSI model is solved and a global optimal algorithm to achieve the optimum EE for any given SE region is proposed. In Section V, the EE-SE tradeoff in the linear RSI model is analyzed. Numerical results are presented in Section VI and the whole paper is concluded in Section VII.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we first describe the system model for FD communications, and then formulate the EE-SE tradeoff problem.

### A. System Model

As depicted in Fig. 1, we consider a TD single picocell network with the small BS (SBS) located at the center of the cell. The SBS has the ability to perform FD communications whereas the user devices work in the conventional HD mode due to their limited hardware capability of SI cancellation. Note that this assumption is also backward compatible with legacy LTE standards.

When SBS is in the FD mode, an uplink user and a downlink user transmit on the same channel simultaneously and are called to be paired as  $U_i$  and  $D_j$  in Fig. 1. Consequently, there are two kinds of interference (see Fig. 1): the SI at the SBS affecting the uplink communication and the CCI at the user side affecting the downlink communication. It is assumed that the SI can be mitigated to a sufficiently low level by applying various SI cancellation techniques. In this paper, we adopt two different RSI models.

- Constant RSI model [9]: the RSI power is assumed to be a constant, denoted by  $\chi$ .
- Linear RSI model [22]: the RSI power is linearly proportional to the transmit power, i.e.,  $\eta P^D$ , where  $\eta$  is a linear coefficient and  $P^D$  is the downlink transmit power.

For both models, it is reasonable to assume that the RSI strength is known to the SBS in advance as it only depends on the results of various SI cancellation techniques. Denote  $h_i^U$  as the uplink carrier-to-noise ratio (CNR) of uplink user  $i$ , which can be expressed as

$$h_i^U = \frac{G\beta_i d_i^{-\alpha}}{N_0}, \quad (1)$$

where  $G$  is the path loss constant,  $\beta_i$  is the fading channel power gain,  $d_i$  is the distance from user  $i$  to the SBS,  $\alpha$  is the path loss exponent, and  $N_0$  is the power of the additive white Gaussian noise (AWGN). Similarly, we can define  $h_j^D$  as the downlink CNR of user  $j$  and  $h_{i,j}$  as the CCI CNR from user  $i$  to user  $j$ . In this paper, we assume that the SBS can acquire the instantaneous channel state information (CSI) of all involved links to perform centralized resource allocation. With this assumption, some insightful results can be obtained in this work. Nevertheless, the SE-EE analysis with imperfect or partial CSI for practical systems deserves further investigation.

### B. Problem Formulation

Consider a picocell with  $M$  pairs of users. As in Fig. 1, uplink user  $i$  and downlink user  $j$  are assumed to be paired, that is, they communicate over the same frequency channel simultaneously. In this case, the achievable uplink and downlink data rates can be expressed as

$$R_{ij}^U = \begin{cases} W \log_2 \left( 1 + \frac{p_{ij}^U h_i^U}{1 + \chi} \right), & \text{for the constant RSI model,} \\ W \log_2 \left( 1 + \frac{p_{ij}^U h_i^U}{1 + \eta p_{ij}^D} \right), & \text{for the linear RSI model,} \end{cases} \quad (2)$$

and

$$R_{ij}^D = W \log_2 \left( 1 + \frac{p_{ij}^D h_j^D}{1 + p_{ij}^U h_{ij}} \right), \quad (3)$$

respectively, where  $W$  is the system bandwidth,  $p_{ij}^U$  and  $p_{ij}^D$  are the uplink and downlink transmit power, respectively. Here, we assume that the constant RSI power,  $\chi$ , and the linear RSI factor,  $\eta$ , are normalized by the noise power  $N_0$ .

The total time-slot length is assumed to be normalized to 1 and let  $\gamma_{ij} \in [0, 1]$  denote the time-slot length allocated to user pair  $(i, j)$ , then the overall system throughput can be expressed as

$$R_{\text{tot}} = \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} (R_{ij}^U + R_{ij}^D). \quad (4)$$

Meanwhile, the total transmit power of uplink and downlink can be expressed as

$$P_{\text{tot}} = \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} (p_{ij}^U + p_{ij}^D). \quad (5)$$

Therefore, for the TD FD small cell network mentioned above, the system EE and system SE can be defined as

$$\beta_{\text{EE}} = \frac{R_{\text{tot}}}{\omega P_{\text{tot}} + P_{\text{fix}}} \quad \text{and} \quad \beta_{\text{SE}} = \frac{R_{\text{tot}}}{W}, \quad (6)$$

respectively, where  $\omega$  represents the inverse of the power amplifier efficiency and  $P_{\text{fix}}$  is the total fixed circuit power consumption of the system, including the power consumption at the transmitter and the receiver.

In this network, the EE-SE tradeoff problem can be formulated as maximizing the system EE for a given system SE, i.e.,  $R_{\text{tot}}$ . The optimization problem can be expressed as

$$\beta_{\text{EE}}^*(R_{\text{tot}}) = \max_{\{R_{ij}^U, R_{ij}^D, \gamma_{ij}\}} \frac{R_{\text{tot}}}{\omega P_{\text{tot}} + P_{\text{fix}}}, \quad (7)$$

$$\text{s.t.} \quad \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} \leq 1, \quad (7a)$$

$$\sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} (R_{ij}^U + R_{ij}^D) = R_{\text{tot}}, \quad (7b)$$

$$\sum_{j=1}^M \gamma_{ij} \geq \gamma_{\min}^U, \quad \forall i, \quad (7c)$$

$$\sum_{i=1}^M \gamma_{ij} \geq \gamma_{\min}^D, \quad \forall j, \quad (7d)$$

where (7a) requires that the overall time-slot length should be no more than the total time-slot length, (7c) and (7d) are the fairness constraints to guarantee a minimum time resource for each uplink user and downlink user, respectively. Here, (7b) indicates that the data rate should be tight at  $R_{\text{tot}}$  since we focus on finding the maximum system EE for a given system SE.

In the following sections, we approach the problem in (7) from three aspects. First, in Section III, the EE-SE tradeoff comparison between FD and HD communications is analyzed and necessary conditions for FD communications to be better than HD communications are derived for both RSI models. Then, the EE-SE tradeoff problem with the constant RSI model is investigated in Section IV. We derive the closed-form expression of the EE-SE tradeoff in the scenario of single pair of users and prove that EE is a quasi-concave function of SE in the scenario of multiple user pairs. Also, the optimal algorithm is developed to achieve the maximum EE based on the quasi-concavity. The linear RSI model is a little bit difficult to analyze, we solve the EE-SE relation numerically in the scenario of single pair of users and propose a heuristic algorithm to achieve the EE-SE tradeoff in the scenario of multiple pairs of users in Section V.

### III. FD OR HD?

Before investigating the EE-SE tradeoff problem for FD networks, it is important to understand when FD communications are better than HD communications in term of EE-SE tradeoff. In this section, necessary conditions are found for FD communications to outperform HD communications for both constant and linear RSI models.

From Section II, the data rates for the FD and the HD communications can be expressed as

$$R_{\text{F}} = R_{ij}^U + R_{ij}^D, \quad (8)$$

and

$$R_{\text{H}} = \max \left( W \log_2 \left( 1 + p_{\text{H}} h_i^U \right), W \log_2 \left( 1 + p_{\text{H}} h_j^D \right) \right), \quad (9)$$

respectively. Note that the goal of this part is to compare the EE between the FD and the HD networks. With the same transmit power, the data rate in (9) is the largest one that the HD communication can achieve. Therefore, it is straightforward to use the maximum data rate for a fair comparison.

Now, the EE comparison problem between FD and HD can be formulated as comparing the EE with the same SE, that is, compare

$$\beta_F = \frac{R_F}{\omega p_F + P_{\text{fix}}} \quad \text{and} \quad \beta_H = \frac{R_H}{\omega p_H + P_{\text{fix}}}, \quad (10)$$

when  $R_F = R_H = R_{\text{tot}}$ , where  $p_F$  and  $p_H$  denote the total transmit powers for the FD and the HD modes, respectively.

Now, the necessary conditions for FD communications to perform better than HD communications can be derived in Appendix A and expressed in the following theorem.

*Theorem 1:* If FD communications are better than HD communications in term of the EE-SE tradeoff, the following conditions must be satisfied

$$h_{ij}(1 + \chi) < \min(h_i^U, h_j^D), \quad \text{for the constant RSI model,} \quad (11)$$

$$\eta + h_{ij} < \min(h_i^U, h_j^D), \quad \text{for the linear RSI model.} \quad (12)$$

From Theorem 1, we have the following intuitive but insightful observations.

- If the CNR of the CCI link,  $h_{ij}$ , is greater than a threshold, i.e.,  $(1 + \chi)^{-1} \min(h_i^U, h_j^D)$  for the constant RSI model and  $\min(h_i^U, h_j^D) - \eta$  for linear RSI model, we should use HD communications rather than FD communications for a better EE-SE tradeoff.
- The FD communications would have a better EE-SE tradeoff than the HD communications only if  $\chi$  is less than the threshold  $h_{ij}^{-1} \min(h_i^U, h_j^D) - 1$  for the constant RSI model or  $\eta$  is less than the threshold  $\min(h_i^U, h_j^D) - h_{ij}$  for the linear RSI model. Otherwise, HD communications should be used because of the large RSI power.

Note that although (11) and (12) in Theorem 1 are not sufficient conditions, they are pretty close to the sufficient conditions when the uplink CNR is close to the downlink CNR. Furthermore, for the linear RSI model, (12) is also close to the sufficient conditions when the transmit power is small. The detailed proof and explanation can be found in Appendix A. Therefore, (11) and (12) will be used as the pre-conditions where the FD mode should be used in the sequel. That is, in the following analysis, it is assumed that (11) is always satisfied in Section IV and (12) is always satisfied in Section V.

#### IV. CONSTANT RSI MODEL

In this section, we focus on the EE-SE tradeoff for the constant RSI model. In this model, we will derive a closed-form EE-SE relation and prove its quasi-concavity.

##### A. Single User Pair

We first start with the scenario of single pair of users to gain some insights of the EE-SE tradeoff. In this case, the

minimum total transmit power can be expressed in a closed-form. Moreover, the maximum EE,  $\beta_{\text{EE}}^*$ , is proved to be a quasi-concave function of the SE, which indicates that the global optimal EE can be achieved for any given SE region.

For a given user pair  $(i, j)$ , the EE-SE tradeoff problem in (7) can be simplified into

$$\beta_{\text{EE}}^*(R_{\text{tot}}) = \max_{\{R_{ij}^U, R_{ij}^D\}} \frac{R_{\text{tot}}}{\omega P_{\text{tot}} + P_{\text{fix}}}, \quad (13)$$

$$\text{s.t. } R_{ij}^U + R_{ij}^D = R_{\text{tot}}. \quad (13a)$$

Obviously, for a given  $R_{\text{tot}}$ , the solution to (13) remains the same if the objective function is replaced by  $P_{ij}^{\text{min}}(R_{\text{tot}}) = \min_{\{R_{ij}^U, R_{ij}^D\}} (p_{ij}^U + p_{ij}^D)$  since the other parameters are constants.

Therefore, we first analyze the minimum transmit power  $P_{ij}^{\text{min}}$  and then investigate the relation between  $\beta_{\text{EE}}^*(R_{\text{tot}})$  and  $R_{\text{tot}}$ . As proved in Appendix B,  $P_{ij}^{\text{min}}(R_{\text{tot}})$  can be solved in a closed-form as

$$P_{ij}^{\text{min}}(R_{\text{tot}}) = \begin{cases} P_1, & \text{if } C_1, \\ P_2, & \text{if } C_2, \\ P_3, & \text{if } \overline{C_1 \& C_2}, \end{cases} \quad (14)$$

where the conditions  $C_1$  and  $C_2$  are defined as

$$\begin{cases} C_1 : A \leq \frac{(h_j^D - h_{ij})(1 + \chi)}{h_i^U - (1 + \chi)h_{ij}}, \\ C_2 : A \leq \frac{h_i^U - (1 + \chi)h_{ij}}{(h_j^D - h_{ij})(1 + \chi)}, \end{cases} \quad (15)$$

and  $P_1$ ,  $P_2$ , and  $P_3$  are defined as

$$\begin{cases} P_1 = (A - 1)/h_j^D, \\ P_2 = (A - 1)(1 + \chi)/h_i^U, \\ P_3 = \frac{2\sqrt{A(1 + \chi)(h_i^U - (1 + \chi)h_{ij})(h_j^D - h_{ij})}}{h_i^U h_j^D} \\ \quad + \frac{(1 + A)(1 + \chi)h_{ij} - h_i^U - (1 + \chi)h_j^D}{h_i^U h_j^D}. \end{cases} \quad (16)$$

In the above,  $A = 2 \frac{R_{\text{tot}}}{W}$ . Note that in (14), in the case of  $C_1$  or  $C_2$ , one user works in the HD mode while the other does not transmit. Therefore, the FD gain lies only in the case of  $\overline{C_1 \& C_2}$ . Furthermore, we can prove in Appendix C that  $P_{ij}^{\text{min}}$  is a monotone convex and strictly increasing function of  $R_{\text{tot}}$ , as presented in the following theorem.

*Theorem 2:* If FD communications are better than HD communications, that is, (11) holds, the minimum transmit power of a pair of FD users in (14) increases with  $R_{\text{tot}}$  and is monotone convex.

According to Theorem 2, we now come up with the following theorem, which is proved in Appendix D.

*Theorem 3:* The maximum EE,  $\beta_{\text{EE}}^*(R_{\text{tot}})$ , is a quasi-concave function of the SE,  $\beta_{\text{SE}}$ , in the scenario of single pair of users.

In [12], the quasi-concavity of EE-SE relation has also been demonstrated in downlink OFDMA networks with HD communications. However, different from the conventional

HD networks, the FD network has two additional kinds of interference: SI at the SBS and CCI at the downlink user side. Besides, the uplink transmission and the downlink transmission are coupled so that the downlink resource and uplink resource should be jointly optimized. These make the EE-SE tradeoff problem in FD networks more complicated than in HD networks. From Theorem 3, under such complicated cases, the EE is still a quasi-concave function of the SE in the scenario of single pair of users.

According to the quasi-concavity in Theorem 3, for a given region of the data rate,  $R_{\text{tot}} \in [R_1, R_2]$ , we can always find the global optimal EE,  $\beta_{\text{EE}}^{**} = \max_{\{R_{\text{tot}}\}} \beta_{\text{EE}}^*(R_{\text{tot}})$ , as follows.

- If  $\beta_{\text{EE}}^*(R_{\text{tot}})$  decreases with  $R_{\text{tot}}$ , i.e.,  $\frac{d\beta_{\text{EE}}^*}{dR_{\text{tot}}} \leq 0$ ,  $\beta_{\text{EE}}^{**}$  is achieved at  $R_1$ .
- If  $\beta_{\text{EE}}^*(R_{\text{tot}})$  increases with  $R_{\text{tot}}$ , i.e.,  $\frac{d\beta_{\text{EE}}^*}{dR_{\text{tot}}} \geq 0$ ,  $\beta_{\text{EE}}^{**}$  is achieved at  $R_2$ .
- If  $\beta_{\text{EE}}^*(R_{\text{tot}})$  first increases and then decreases,  $\beta_{\text{EE}}^{**}$  is achieved at  $R^*$ , where

$$\frac{d\beta_{\text{EE}}^*}{dR_-^*} \geq 0 \text{ and } \frac{d\beta_{\text{EE}}^*}{dR_+^*} \leq 0. \quad (17)$$

### B. Multiple User Pairs

In this section, we extend the EE-SE tradeoff analysis into the scenario of multiple user pairs. We also show that the EE is a quasi-concave function of the SE in this scenario. Based on the quasi-convexity, the global optimal algorithm to achieve the maximum EE for any given SE region will be developed.

Similar to Section IV-A, we first consider maximizing the EE for a given system SE, i.e.,  $R_{\text{tot}}$ . In the multi-user scenario, the objective function of (7) can be similarly expressed as

$$P_{\text{tot}}^{\min} = \min_{\{R_{ij}^U, R_{ij}^D\}} \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} P_{ij}, \quad (18)$$

where  $P_{ij} = \min_{\{R_{ij}^U, R_{ij}^D\}} (p_{ij}^U + p_{ij}^D)$  is the minimum total transmit power of user pair  $(i, j)$ .

Given the data rate of user pair  $(i, j)$  as  $R_{ij} = R_{ij}^U + R_{ij}^D$ ,  $P_{ij} = P_{ij}^{\min}(R_{ij})$  can be expressed in (14). Substituting  $P_{ij}$  into (18) leads to a non-convex optimization problem due to the non-convexity of  $\gamma_{ij} P_{ij}$  and  $\gamma_{ij} R_{ij}$ . In the following, we will first transform the problem into a convex one.

We define an auxiliary variable, as

$$\hat{R}_{ij} = \gamma_{ij} R_{ij}. \quad (19)$$

Then, by substituting it into (18), the EE maximization problem can be transformed into

$$P_{\text{tot}}^{\min} = \min_{\{\gamma_{ij}, \hat{R}_{ij}\}} \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} P_{ij}, \quad (20)$$

$$\text{s.t. } \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} \leq 1, \quad (20a)$$

$$\sum_{i=1}^M \sum_{j=1}^M \hat{R}_{ij} = R_{\text{tot}}, \quad (20b)$$

$$\sum_{j=1}^M \gamma_{ij} \geq \gamma_{\min}^U, \quad \forall i, \quad (20c)$$

$$\sum_{i=1}^M \gamma_{ij} \geq \gamma_{\min}^D, \quad \forall j. \quad (20d)$$

TABLE I

OPTIMAL METHOD TO FIND THE GLOBAL OPTIMUM EE FOR A GIVEN REGION OF  $\beta_{\text{SE}}$

**Algorithm 1** The optimal method to find the global optimum EE for a given region of  $\beta_{\text{SE}}$

- 1: Let  $R_b = R_1$ ,  $R_e = R_2$  and  $\delta > 0$  as the tolerance.
- 2: Solve the inner loop and calculate  $\beta_{\text{EE}}^*(R_b)$  and  $\beta_{\text{EE}}^*(R_e)$ .
- 3: **Loop**
- 4:  $R_m = (R_b + R_e)/2$ .
- 5: Solve the inner loop and calculate  $\beta_{\text{EE}}^*(R_m)$ .
- 6: **if**  $\beta_{\text{EE}}^*(R_m) > \beta_{\text{EE}}^*(R_b)$
- 7:  $R_b = R_m$ .
- 8: **else**
- 9:  $R_e = R_m$ .
- 10: **Until**  $|\beta_{\text{EE}}^*(R_b) - \beta_{\text{EE}}^*(R_e)| < \delta$ .
- 11:  $\beta_{\text{EE}}^{**} = \beta_{\text{EE}}^*(R_b)$ .

As shown in Appendix E, the optimization problem in (20) is now convex with regard to the variables  $\Gamma = \{\gamma_{ij}\}_M$  and  $\hat{\mathbf{R}} = \{\hat{R}_{ij}\}_M$ , when  $\gamma_{ij} \in (0, 1]$ . Based on the convexity of the EE maximization problem in (20), we can further show that the EE is a quasi-concave function of the SE in Appendix F, as presented in the following theorem.

*Theorem 4:* The optimal EE,  $\beta_{\text{EE}}^*(R_{\text{tot}})$ , is a quasi-concave function of the SE,  $\beta_{\text{SE}}$ , in the scenario of multiple user pairs.

Therefore, the global optimum EE,  $\beta_{\text{EE}}^{**} = \max_{\{R_{\text{tot}}\}} \beta_{\text{EE}}^*(R_{\text{tot}})$ , can be achieved for any given SE region, as elaborated in the following.

- *The inner loop:* For a fixed SE, i.e.,  $R_{\text{tot}}$ , solve the convex problem in (20) to obtain  $\beta_{\text{EE}}^*(R_{\text{tot}}) = \max_{\{\gamma_{ij}, \hat{R}_{ij}\}} \frac{R_{\text{tot}}}{\omega P_{\text{tot}} + P_{\text{fix}}}$ .
- *The outer loop:* Find the global optimum EE,  $\beta_{\text{EE}}^{**}$ , within the SE region.

The detailed procedures to achieve  $\beta_{\text{EE}}^{**}$  for a given region of  $R_{\text{tot}} \in [R_1, R_2]$  are illustrated in Table I. In the table, the outer loop can be solved by the bisection method [24], whose computational complexity is  $O(\frac{1}{\delta})$ , where  $\delta$  is the convergence tolerance. The inner loop is a convex optimization problem, which can be solved with a computational complexity of  $O(N^3)$ , where  $N = 2M^2$  is the size of the convex problem. Therefore, the overall computational complexity of Algorithm 1 is  $O(\frac{1}{\delta} N^3)$ .

In what follows, we solve the convex problem in the inner loop using the Lagrangian duplex decomposition (LDD) technique [25]. The Lagrangian function of (20) can be written as

$$\begin{aligned} \mathcal{L}(\Gamma, \hat{\mathbf{R}}, \lambda, \tau, \boldsymbol{\alpha}, \boldsymbol{\mu}) &= \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} P_{ij} \\ &+ \lambda (\sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} - 1) \\ &+ \tau (\sum_{i=1}^M \sum_{j=1}^M \hat{R}_{ij} - R_{\text{tot}}) \end{aligned}$$

$$\begin{aligned}
& - \sum_{i=1}^M \alpha_i \left( \sum_{j=1}^M \gamma_{ij} - \gamma_{\min}^U \right) \\
& - \sum_{j=1}^M \mu_j \left( \sum_{i=1}^M \gamma_{ij} - \gamma_{\min}^D \right), \quad (21)
\end{aligned}$$

where  $\lambda$ ,  $\tau$ ,  $\alpha$ , and  $\mu$  are the Lagrangian multiplier vectors.

Therefore, the dual problem is

$$\max_{\{\lambda, \tau, \alpha, \mu\}} \min_{\{\Gamma, \hat{\mathbf{R}}\}} \mathcal{L}(\Gamma, \hat{\mathbf{R}}, \lambda, \tau, \alpha, \mu), \quad (22)$$

which can be decomposed into two layers: the inner layer to minimize  $\mathcal{L}$  for given multiplier vectors and the outer layer to maximize the master dual problem over  $\lambda$ ,  $\tau$ ,  $\alpha$ , and  $\mu$ .

1) *inner layer*: For given  $\lambda$ ,  $\tau$ ,  $\alpha$ , and  $\mu$ , the inner layer minimization problem can be solved by the subgradient method [26] as

$$\begin{cases} \gamma_{ij}(k+1) = \min \left( \gamma_{ij}(k) - \epsilon \frac{\partial \mathcal{L}}{\gamma_{ij}}, 1 \right), \\ \hat{R}_{ij}(k+1) = \max \left( \hat{R}_{ij}(k) - \zeta \frac{\partial \mathcal{L}}{\hat{R}_{ij}}, 0 \right), \end{cases} \quad (23)$$

where  $\epsilon$  and  $\zeta$  are sufficiently small step sizes.

2) *outer layer*: Once the inner layer is solved, the solution to the dual problem can be updated by

$$\begin{cases} \lambda(k+1) = \max \left( \lambda(k) + \phi \left( \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} - 1 \right), 0 \right), \\ \tau(k+1) = \tau(k) + \psi \left( \sum_{i=1}^M \sum_{j=1}^M \hat{R}_{ij} - R_{\text{tot}} \right), \\ \alpha_i(k+1) = \max \left( \alpha_i(k) - \zeta_i \left( \sum_{j=1}^M \gamma_{ij}^U - \gamma_{\min}^U \right), 0 \right), \\ \mu_j(k+1) = \max \left( \mu_j(k) - \kappa_j \left( \sum_{i=1}^M \gamma_{ij}^D - \gamma_{\min}^D \right), 0 \right), \end{cases} \quad (24)$$

where  $\phi$ ,  $\psi$ ,  $\zeta_i$ , and  $\kappa_j$  are sufficiently small step sizes.

## V. LINEAR RSI MODEL

So far, we have proved the quasi-concavity of the EE-SE relation and proposed a global optimal solution to obtain the maximum EE with the constant RSI model. In this section, we further extend our analysis into the linear RSI model, where the RSI power is linearly proportional to the transmit power.

### A. Single User Pair

First, we analyze the EE-SE tradeoff in the scenario of single pair of users. The EE-SE tradeoff problem in this case is similar to (13) in Section IV. Likewise, the objective function can be replaced by  $P_{ij}^{\min} = \min_{\{R_{ij}^U, R_{ij}^D\}} (p_{ij}^U + p_{ij}^D)$ . However, in this model, the constraint in (13a) can be expressed as

$$\begin{aligned}
& p_{ij}^U h_i^U h_{ij} + p_{ij}^U p_{ij}^D [(1-A)\eta h_{ij} + h_i^U h_j^D] + p_{ij}^D \eta h_j^D \\
& + p_{ij}^U [(1-A)h_{ij} + h_i^U] + p_{ij}^D [h_j^D + (1-A)\eta] \\
& + 1 - A = 0. \quad (25)
\end{aligned}$$

With some simple mathematical analysis, (25) turns out to be a conic curve with regard to  $p_{ij}^U$  and  $p_{ij}^D$ . Therefore, the optimal solution to (13) lies in one of the following three points: 1)  $B_1$  where  $\frac{dp_{ij}^D}{dp_{ij}^U} = -1$ , 2)  $B_2$  where  $p_{ij}^U = 0$ , and 3)

$B_3$  where  $p_{ij}^D = 0$ . Obviously, it is easy to solve  $B_2$  and  $B_3$ . In the following, we will solve  $B_1$ .

By substituting  $\frac{dp_{ij}^D}{dp_{ij}^U} = -1$ , the first derivative of (25) can be expressed as

$$\begin{aligned}
& 2p_{ij}^U h_i^U h_{ij} + p_{ij}^D [(1-A)\eta h_{ij} + h_i^U h_j^D] \\
& - p_{ij}^U [(1-A)\eta h_{ij} + h_i^U h_j^D] - 2p_{ij}^D \eta h_j^D \\
& + [(1-A)h_{ij} + h_i^U] - [h_j^D + (1-A)\eta] = 0. \quad (26)
\end{aligned}$$

By jointly considering (25) and (26), the point  $B_1$  can be obtained numerically. Finally, by comparing  $B_1$ ,  $B_2$ , and  $B_3$ , the optimal solution can be achieved as  $\min\{B_1, B_2, B_3\}$ .

### B. Multiple User Pairs

Recall that the EE-SE tradeoff problem with the linear RSI model can be expressed in (7). Likewise, for a given SE, the problem can be equivalently converted into the power minimization problem, i.e., the solution remains the same by replacing the objective function with  $P_{\text{tot}}^{\min} = \min_{\{\gamma_{ij}, R_{ij}^U, R_{ij}^D\}} \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} P_{ij}$ . However, in this model, the EE maximization problem is far more complicated since the closed-form solution to  $P_{ij}^{\min}$  can not be obtained as well as the quasi-convexity of the EE-SE relation can not be easily proved. Therefore, in the following, we solve the EE maximization problem heuristically by decoupling it into two sub-problems, namely, power control and resource allocation.

1) *Power control*: In the power control step, the total data of each FD pair is fixed to  $R_{\text{tot}}$ . Therefore, the corresponding transmit power of each FD pair can be solved numerically as in Section V-A.

2) *Resource allocation*: In this step, the constraint in (7a) is further bounded to  $\sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} = 1$ . Therefore, the SE constraint in (7b) can be always satisfied. The problem turns out to be a linear programming as

$$P_{\text{tot}}^{\min} = \min_{\{\gamma_{ij}\}} \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} P_{ij}, \quad (27)$$

$$\text{s.t. } \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} = 1, \quad (27a)$$

$$\sum_{j=1}^M \gamma_{ij} \geq \gamma_{\min}^U, \quad \forall i, \quad (27b)$$

$$\sum_{i=1}^M \gamma_{ij} \geq \gamma_{\min}^D, \quad \forall j, \quad (27c)$$

which can be effectively solved by the simplex method [27].

The detailed procedures of the proposed heuristic algorithm are summarized in Table II, which generally has a polynomial complexity. As mentioned above, it is very difficult to achieve the optimal solution in this model. However, the performance of the proposed heuristic algorithm can be guaranteed for the following two reasons. First, the optimal solution in the scenario of single pair of users can be solved numerically, which is then being used to solve the problem in the scenario of multiple pairs of users. Besides, in the algorithm, it is guaranteed that all time-slot resource can be fully utilized according to Line 3 in Algorithm 2.

TABLE II  
HEURISTIC ALGORITHM FOR THE LINEAR RSI MODEL

**Algorithm 2** The heuristic algorithm for the linear RSI model

- 1: Let  $R_{ij} = R_{\text{tot}}, \forall i, j$ .
- 2:  $P_{ij} = \min(B_1, B_2, B_3)$ .
- 3: Bound (7a) into  $\sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} = 1$ .
- 4: Replace the objective function of (7) by  $P_{\text{tot}}^{\min} = \min_{\{\gamma_{ij}\}} \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} P_{ij}$ .
- 5: Solve (27) using the simplex method.

TABLE III  
SIMULATION PARAMETERS

Parameter	Value
Cell radius, $r$	150 m
Bandwidth, $W$	10 MHz
Noise power density, $N_0$	-174 dBm/Hz
Path loss between user and SBS	$145.4 + 37.5 \log(d(\text{km}))$
SSD between user and SBS	10 dB
Path loss between users, if $d < 0.05$	$98.45 + 20 \log(d(\text{km}))$
Path loss between users, if $d \geq 0.05$	$175.78 + 40 \log(d(\text{km}))$
SSD between users	12 dB

## VI. NUMERICAL RESULTS

In the simulation, we consider a single small cell network with a radius of 150 m. The FD enabled SBS is located at the center of the cell. User devices are uniformly distributed in the cell. The parameters of pathloss fading and *shadow standard deviation* (SSD) are according to [28]. The major simulation parameters are listed in Table III.

### A. Constant RSI Model

We first show the results for the constant RSI model. Fig. 2 compares the EE-SE tradeoff of FD and HD communications in the scenario of single pair of users. In the figure,  $\chi_1$ ,  $\chi_2$ , and  $\chi_3$  are the critical points calculated from the necessary condition in (11) for  $h_{ij} = 30$ , 20, and 10, respectively. As we can see from the figure, they are larger than the exact intersection points where FD communications would be better than HD communications. This shows that (11) is actually a necessary condition and therefore verifies Theorem 1.

Fig. 3 presents the relation of the maximum EE,  $\beta_{\text{EE}}^*(R_{\text{tot}})$ , and the SE,  $\beta_{\text{SE}}$ , when there are six pairs of users, i.e.,  $M = 6$ . From the figure, the minimum total transmit power,  $P_{\text{tot}}^{\min}$ , increases with the SE,  $\beta_{\text{SE}}$ , in both FD and HD networks. For the FD network, the larger the RSI power  $\chi$  is, the more rapidly the transmit power increases. This is because that more transmit power is needed to surpass the RSI power to achieve the same SE when  $\chi$  is large. In Fig. 3(b), in both FD mode and HD mode,  $\beta_{\text{EE}}^*$  first increases and then decreases with the SE, which indicates that the maximum EE,  $\beta_{\text{EE}}^*$ , is a quasi-concave function of the SE,  $\beta_{\text{SE}}$ , and validates Theorem 4.

Fig. 4 plots the maximum EE,  $\beta_{\text{EE}}^*$ , for different numbers of user pairs,  $M$ . From the figure,  $\beta_{\text{EE}}^*$  decreases with the number of user pairs for the HD network. This is because that

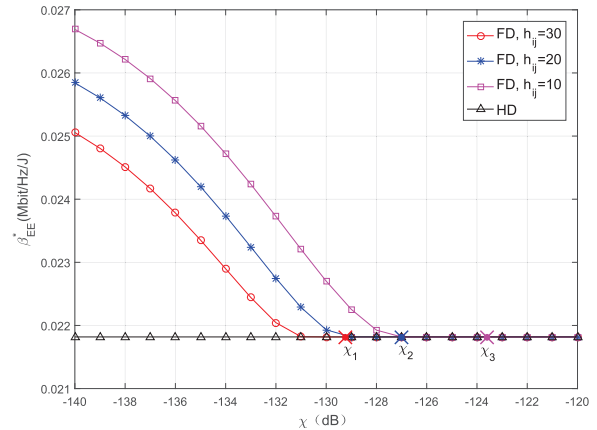
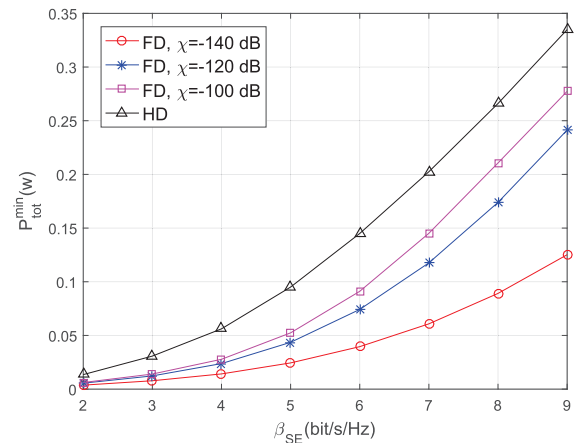
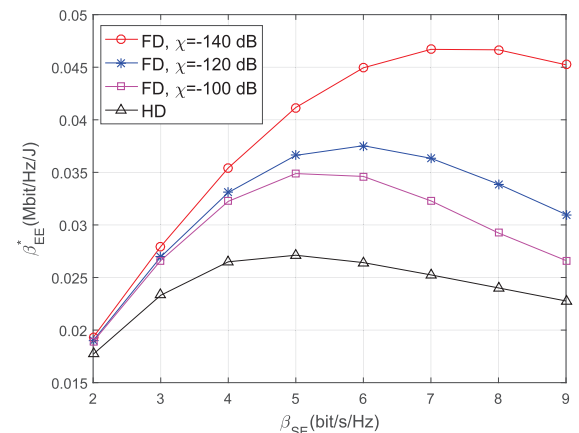


Fig. 2. EE-SE tradeoff comparison in the scenario of single pair of users for the constant RSI model.  $\beta_{\text{SE}} = 4$  bit/s/Hz,  $h_i^U = 120$ ,  $h_j^D = 180$ .



(a) The minimum transmit power,  $P_{\text{tot}}^{\min}$ , with the SE,  $\beta_{\text{SE}}$ .



(b) The maximum EE,  $\beta_{\text{EE}}^*$ , with the SE,  $\beta_{\text{SE}}$ .

Fig. 3. Relation of the maximum EE,  $\beta_{\text{EE}}^*$ , and the SE,  $\beta_{\text{SE}}$ , for the constant RSI model.  $M = 6$ .

more resource needs to be allocated to guarantee the fairness constraints in (7c) and (7d) and more power will be radiated as the number of users increases, leading to the degradation of EE. However, in the FD mode,  $\beta_{\text{EE}}^*$  increases with the number of user pairs because more users lead to more opportunities for pairing users.

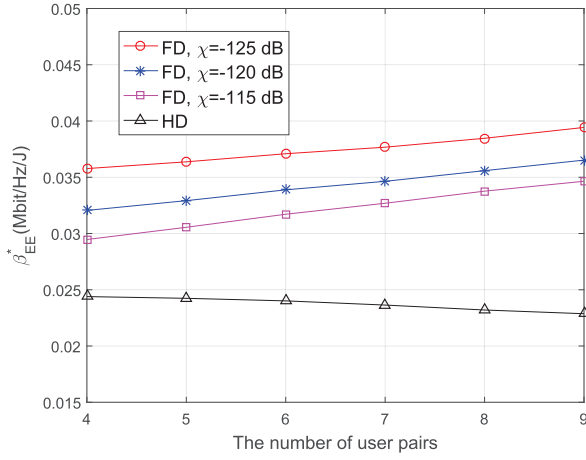


Fig. 4. EE-SE tradeoff with different numbers of user pairs for the constant RSI model.  $\beta_{SE} = 8$  bit/s/Hz.

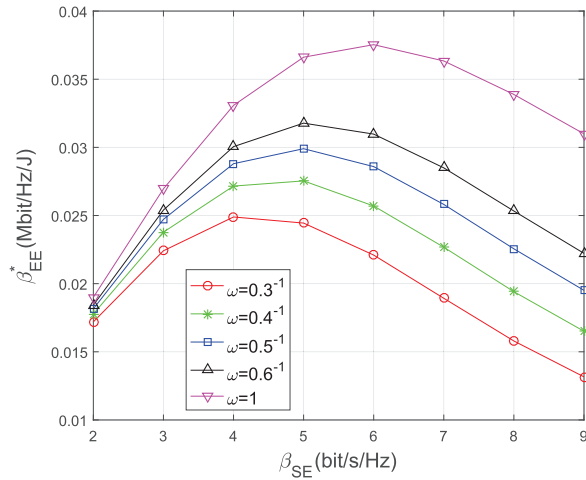


Fig. 5. EE-SE relation with different  $\omega$ .  $\chi = -120$  dB.

To further demonstrate the impact of the power amplifier efficiency, we present the EE-SE relation with different  $\omega$  in Fig. 5. From the figure, when  $\beta_{SE}$  is small, the value of  $\omega$  has little impact on  $\beta_{EE}^*$  since the total transmit power is relatively small in this case. Note that with different  $\omega$ , the EE-SE relation still remains quasi-concave, which validates that the value of  $\omega$  does not influence the property of the EE-SE relation.

### B. Linear RSI Model

In this section, the numerical results of the linear RSI model are shown. Fig. 6 compares the FD and the HD communications in term of EE-SE tradeoff in the scenario of single pair of users. In the figure,  $\eta_1$ ,  $\eta_2$ , and  $\eta_3$  are the critical points calculated from the necessary condition (12) for  $h_{ij} = 30$ , 20, and 10, respectively. Similar to the constant RSI model, they are larger than the exact intersection points where the performance of FD communications would be better than HD communications. This validates the necessary condition (12) for the linear RSI model in Theorem 1.

Fig. 7 shows the relation between the maximum EE,  $\beta_{EE}^*$ , and the SE,  $\beta_{SE}$ , for the linear RSI model. Similar to the constant RSI model, in both FD and HD networks, the

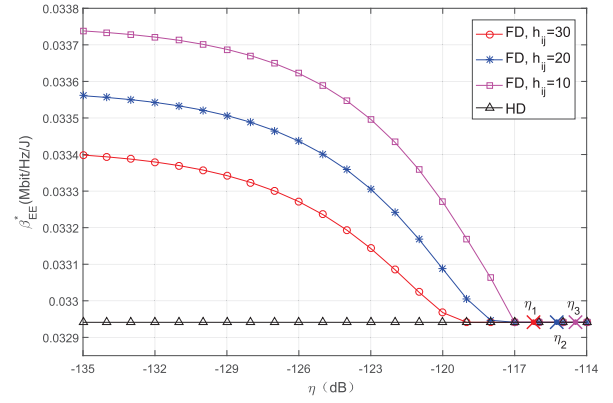
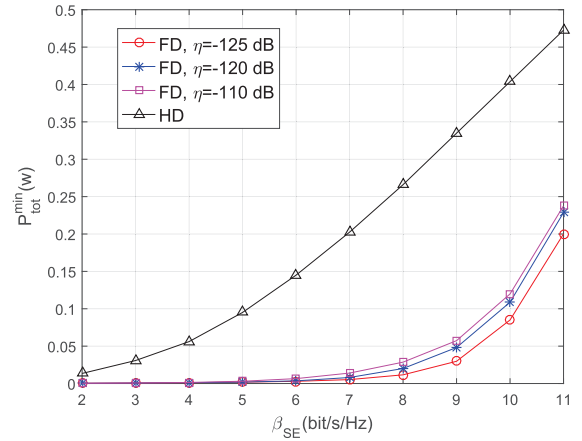
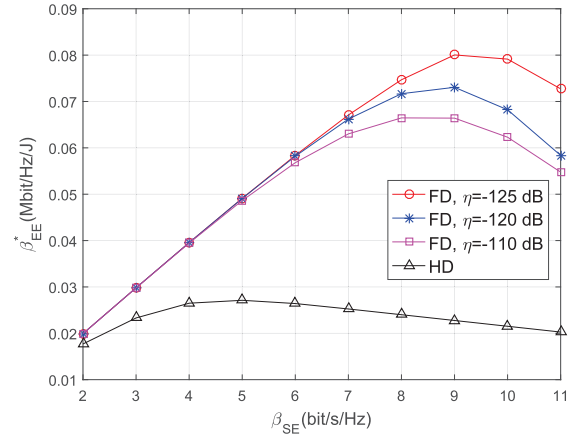


Fig. 6. EE-SE tradeoff comparison in the scenario of single pair of users for the linear RSI model.  $\beta_{SE} = 4$  bit/s/Hz,  $h_i^U = 700$ ,  $h_j^D = 120$ .



(a) The total transmit power,  $P_{tot}$ , with the SE,  $\beta_{SE}$ .



(b) The maximum EE,  $\beta_{EE}^*$ , with the SE,  $\beta_{SE}$ .

Fig. 7. Relation of the maximum EE,  $\beta_{EE}^*$ , and the SE,  $\beta_{SE}$ , for the linear RSI model.  $M = 6$ .

minimum total transmit power,  $P_{tot}^{\min}$ , increases with the SE,  $\beta_{SE}$ . Moreover,  $\beta_{EE}^*$  first increases and then decreases with the  $\beta_{SE}$ , which indicates that  $\beta_{EE}^*$  is also a quasi-concave function of  $\beta_{SE}$  for the linear RSI model.

The maximum EE,  $\beta_{EE}^*$ , for different numbers of user pairs,  $M$ , for the linear RSI model is depicted in Fig. 8. From the figure,  $\beta_{EE}^*$  increases with  $M$  in the FD network while it slightly decreases in the HD network. This phenomenon is similar to Fig. 4.



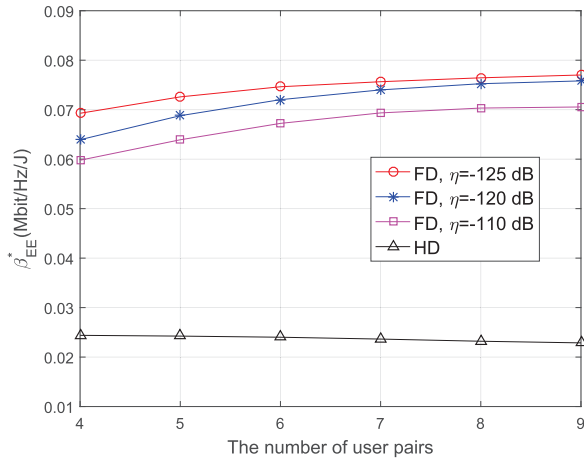


Fig. 8. EE-SE tradeoff with different numbers of user pairs for the constant RSI model.  $\beta_{SE} = 8$  bit/s/Hz.

## VII. CONCLUSION AND DISCUSSION

In this paper, we have investigated the fundamental EE-SE tradeoff for FD enabled cellular networks where the base stations work in the FD mode while the user devices still work in the conventional HD mode. Two different RSI models are considered, namely, the constant RSI model and the linear RSI model. We have first addressed the problem of whether to use the FD mode and derived necessary conditions for the FD mode achieves a better EE-SE tradeoff than the HD mode for both RSI models. Then, for the constant RSI model, we have proved that the EE is a quasi-concave function of the SE and derived a closed-form EE expression in the scenario of single pair of users. Thereby, the global optimal solution to achieve the maximum EE for any given region of SE has been developed. For the linear RSI model, due to the complicated EE expression, we have solved the EE-SE tradeoff problem numerically and proposed a heuristic algorithm to achieve the maximum EE. Extensive numerical results have confirmed our analysis and demonstrated the effectiveness of our algorithms.

The significance of this paper is revealing that the EE-SE relation in a cellular network with FD enabled BS is quasi-concave. With this property, it is possible for the industry to design the optimal EE-oriented resource allocation strategy while guaranteeing a given required SE. Moreover, in our paper, we have considered two different RSI models. In fact, the self-interference cancellation techniques are still in its infancy. It is unknown whether the RSI is related to the transmit power or not. Therefore, we have to assume two different models for the completeness of our study. Fortunately, we find that the quasi-concave EE-SE relation can be achieved for both RSI models.

### APPENDIX A

To prove Theorem 1, we first consider the case where the data rate of the FD mode is less than that of the HD mode for a given transmit power, as

$$R_F \leq R_H, \forall p_{ij}^U + p_{ij}^D = p_H. \quad (28)$$

For the constant RSI model, by substituting  $R_F$  in (8) and  $R_H$  in (9) into (28), it can be further derived as

$$\begin{aligned} & \left(1 + \frac{p_{ij}^U h_i^U}{1 + \chi}\right) \left(1 + \frac{p_{ij}^D h_j^D}{1 + p_{ij}^U h_{ij}}\right) \\ & \leq \max(1 + p_H h_i^U, 1 + p_H h_j^D), \forall p_{ij}^U + p_{ij}^D = p_H. \end{aligned} \quad (29)$$

Then, by expanding the left-hand side of (29), we obtain

$$\begin{aligned} & \frac{p_{ij}^U h_i^U}{1 + \chi} + \frac{p_{ij}^D h_j^D}{1 + p_{ij}^U h_{ij}} + \frac{p_{ij}^U h_i^U p_{ij}^D h_j^D}{(1 + \chi)(1 + p_{ij}^U h_{ij})} \\ & \leq \max(p_H h_i^U, p_H h_j^D), \forall p_{ij}^U + p_{ij}^D = p_H. \end{aligned} \quad (30)$$

Without loss of generality,  $h_i^U$  is assumed to be no less than  $h_j^D$ . Therefore, by relaxing  $h_j^D$  to  $h_i^U$  in the second term of the left-hand side, we have

$$\begin{aligned} & \frac{p_{ij}^U h_i^U}{1 + \chi} + \frac{p_{ij}^D h_i^U}{1 + p_{ij}^U h_{ij}} + \frac{p_{ij}^U h_i^U p_{ij}^D h_j^D}{(1 + \chi)(1 + p_{ij}^U h_{ij})} \\ & \leq p_H h_i^U, \forall p_{ij}^U + p_{ij}^D = p_H. \end{aligned} \quad (31)$$

which is a sufficient condition for (28). When  $h_i^U$  and  $h_j^D$  are close, this relaxation is tight. Moreover, if  $h_i^U = h_j^D$ , (31) is equivalent to (30).

Moreover, by dividing  $h_i^U$  and multiplying  $(1 + \chi)(1 + p_{ij}^U h_{ij})$  at both sides, (31) can be further derived as

$$\begin{aligned} & p_{ij}^U (1 + p_{ij}^U h_{ij}) + p_{ij}^D (1 + \chi) + p_{ij}^U p_{ij}^D h_j^D \\ & \leq p_H (1 + \chi)(1 + p_{ij}^U h_{ij}), \quad \forall p_{ij}^U + p_{ij}^D = p_H. \end{aligned} \quad (32)$$

Next, by substituting  $p_{ij}^U + p_{ij}^D = p_H$  into (32) and then subtracting  $p_{ij}^U (1 + p_{ij}^U h_{ij}) + p_{ij}^D (1 + \chi)$  from both sides, it can be further derived as

$$\begin{aligned} & p_{ij}^U p_{ij}^D h_j^D \leq p_{ij}^U \chi (1 + p_{ij}^U h_{ij}) \\ & \quad + p_{ij}^D p_{ij}^U h_{ij} (1 + \chi), \quad \forall p_{ij}^U + p_{ij}^D = p_H. \end{aligned} \quad (33)$$

Then, by dividing  $p_{ij}^U p_{ij}^D$  at both sides and rearranging the terms, we have

$$\left(\frac{\chi}{p_{ij}^D} + h_{ij}\right) + (p_{ij}^U + p_{ij}^D) \frac{\chi}{p_{ij}^D} h_{ij} \geq h_j^D, \forall p_{ij}^U + p_{ij}^D = p_H. \quad (34)$$

Therefore, when

$$h_{ij} (1 + \chi) \geq h_j^D, \quad (35)$$

(28) is always satisfied, i.e., for all transmit power,  $R_H \geq R_F$ .

Similarly, if  $h_j^D > h_i^U$ , a similar condition in (35) by replacing  $h_j^D$  with  $h_i^U$  can be achieved. Therefore, the sufficient condition for (28) can be given as

$$h_{ij} (1 + \chi) \geq \min(h_i^U, h_j^D). \quad (36)$$

For the linear RSI model, by replacing  $\chi$  with  $\eta p_{ij}^D$  in (30) and (31), a sufficient condition for (28) can be derived as

$$\eta + h_{ij} + (p_{ij}^U + p_{ij}^D) \eta h_{ij} \geq \min(h_i^U, h_j^D). \quad (37)$$

Furthermore, by neglecting the term  $(p_{ij}^U + p_{ij}^D) \eta h_{ij}$  in the left-hand side of (37), a sufficient condition of (28) can be derived as

$$\eta + h_{ij} \geq \min(h_i^U, h_j^D). \quad (38)$$

Note that when  $p_{ij}^U + p_{ij}^D$  is small, (38) is close to (37).

We are now ready to prove Theorem 1. Assuming that  $R_F = R_H = R$  and the minimum transmit power of FD mode and HD mode are  $p_F$  and  $p_H$ , respectively. Let  $R_H^*$  be the data rate in the HD mode when the total transmit power is  $p_F$ . If (36) is satisfied for the constant RSI model or (38) is satisfied for the linear RSI model, we have  $R_H^* \geq R_F = R_H$ . Moreover, since the achievable data rate strictly increases with the transmit power in the HD mode, we have  $p_F \geq p_H$ . This ends the proof.

#### APPENDIX B

By substituting  $R_{ij}^U$  in (2) and  $R_{ij}^D$  in (3) into (13a), we can derive that

$$W \log_2\left(1 + \frac{p_{ij}^U h_i^U}{1 + \chi}\right) + W \log_2\left(1 + \frac{p_{ij}^D h_j^D}{1 + p_{ij}^U h_{ij}}\right) = R_{\text{tot}}. \quad (39)$$

Note that the optimization variables are now changed into  $p_{ij}^U$  and  $p_{ij}^D$ . Define  $A = 2^{\frac{R_{\text{tot}}}{W}}$ , we have

$$\left(1 + \frac{p_{ij}^U h_i^U}{1 + \chi}\right)\left(1 + \frac{p_{ij}^D h_j^D}{1 + p_{ij}^U h_{ij}}\right) = A. \quad (40)$$

Then, by multiplying  $(1 + \chi)(1 + p_{ij}^U h_{ij})$  at both sides, we can further derive that

$$\begin{aligned} & (1 + \chi + p_{ij}^U h_i^U)(1 + p_{ij}^U h_{ij} + p_{ij}^D h_j^D) \\ & = A(1 + \chi)(1 + p_{ij}^U h_{ij}). \end{aligned} \quad (41)$$

Next, by expanding the above equation and merging the similar terms, it can be written into

$$\begin{aligned} & p_{ij}^U{}^2 h_i^U h_{ij} + p_{ij}^U p_{ij}^D h_i^U h_j^D \\ & + p_{ij}^U [(1 - A)(1 + \chi)h_{ij} + h_i^U] + (1 + \chi)p_{ij}^D h_j^D \\ & + (1 - A)(1 + \chi) = 0. \end{aligned} \quad (42)$$

Furthermore, the relation between  $p_{ij}^U$  and  $p_{ij}^D$  in (42) forms a conic curve with the solution to  $P_{ij}^{\min}$  locating at  $\frac{dp_{ij}^D}{dp_{ij}^U} = -1$ .

Hence, by substituting  $\frac{dp_{ij}^D}{dp_{ij}^U} = -1$  into (42), we can further derive

$$\begin{aligned} & p_{ij}^U (h_i^U h_j^D - 2h_i^U h_{ij}) + h_j^D (1 + \chi) \\ & - (1 - A)(1 + \chi)h_{ij} - h_i^U - p_{ij}^D h_i^U h_j^D = 0. \end{aligned} \quad (43)$$

In what follows, we will solve (13) by jointly considering (42) and (43). First, we rewrite (43) as

$$\begin{aligned} p_{ij}^D & = \frac{p_{ij}^U (h_i^U h_j^D - 2h_i^U h_{ij}) + h_j^D (1 + \chi)}{h_i^U h_j^D} \\ & - \frac{(1 - A)(1 + \chi)h_{ij} + h_i^U}{h_i^U h_j^D}. \end{aligned} \quad (44)$$

Substituting (44) into (42) and assuming  $h_{ij}(1 + \chi) < \min(h_i^U, h_j^D)$ ,  $p_{ij}^U$  and  $p_{ij}^D$  can be solved as  $p_*^U$  and  $p_*^D$ ,

respectively, where

$$p_*^U = \frac{1}{h_i^U} \left( \sqrt{\frac{A(1 + \chi)(h_i^U - (1 + \chi)h_{ij})}{h_j^D - h_{ij}}} - (1 + \chi) \right), \quad (45)$$

and

$$\begin{aligned} p_*^D & = \frac{\sqrt{A(1 + \chi)(h_i^U - (1 + \chi)h_{ij})(h_j^D - 2h_{ij})}}{h_i^U h_j^D \sqrt{h_j^D - h_{ij}}} \\ & + \frac{(1 + A)(1 + \chi)h_{ij} - h_i^U}{h_i^U h_j^D}. \end{aligned} \quad (46)$$

Note that  $p_*^U$  in (45) and  $p_*^D$  in (46) might be less than zero. Therefore, we shall discuss the following three cases: 1)  $C_1 : p_*^U < 0$ , 2)  $C_2 : p_*^D < 0$ , and 3)  $\overline{C_1} \& \overline{C_2}$ , and solve the optimal  $P_{ij}^{\min}$  for each case.

According to (45) and (46),  $C_1$  and  $C_2$  can be simplified into

$$C_1 : A < \frac{(h_j^D - h_{ij})(1 + \chi)}{h_i^U - (1 + \chi)h_{ij}}, \quad (47)$$

and

$$C_2 : A < \frac{h_i^U - (1 + \chi)h_{ij}}{(h_j^D - h_{ij})(1 + \chi)}, \quad (48)$$

respectively. We can observe that  $C_1 \& C_2 = \phi$ . Therefore, we have  $C_1 = C_1 \& \overline{C_2}$  and  $C_2 = C_2 \& \overline{C_1}$ . Moreover,  $C_1 \cup C_2 \cup (\overline{C_1} \& \overline{C_2})$  is the whole solution space of  $(p_{ij}^U, p_{ij}^D)$ . Furthermore, it can be derived that the solution space of  $(p_{ij}^U, p_{ij}^D)$  is  $C_1 \cup (\overline{C_1} \& \overline{C_2})$  or  $C_2 \cup (\overline{C_1} \& \overline{C_2})$ .

Now, we solve the  $P_{ij}^{\min}$  for each case.

1) In the case of  $C_1$ ,  $p_{ij}^U$  and  $p_{ij}^D$  can be solved as

$$\begin{cases} p_{ij}^U = 0, \\ p_{ij}^D = \frac{A - 1}{h_j^D}, \end{cases} \quad (49)$$

and the corresponding  $P_{ij}^{\min}$  can be expressed as

$$P_{ij}^{\min} = \frac{A - 1}{h_j^D}. \quad (50)$$

2) In the case of  $C_2$ ,  $p_{ij}^U$  and  $p_{ij}^D$  can be solved as

$$\begin{cases} p_{ij}^U = \frac{(A - 1)(1 + \chi)}{h_i^U}, \\ p_{ij}^D = 0, \end{cases} \quad (51)$$

and the corresponding  $P_{ij}^{\min}$  can be expressed as

$$P_{ij}^{\min} = \frac{(A - 1)(1 + \chi)}{h_i^U}. \quad (52)$$

3) In the case of  $\overline{C_1} \& \overline{C_2}$ ,  $p_{ij}^U$  and  $p_{ij}^D$  can be solved in (45) and (46), respectively.

In summary, the optimal  $P_{ij}^{\min}$  can be expressed as (14).

## APPENDIX C

For notation simplicity, we use  $R$  as  $R_{\text{tot}}$  in the sequel. It can be easily shown that  $P_{ij}^{\min}$  is convex and increases with  $R$  since  $2^R$  strictly increases with  $R$ . We now prove  $P_k, k = 1, 2, 3$ , defined in (16), are convex. Since  $2^R$  is strictly convex on  $R$  and affine transformation preserves convexity [24],  $P_1$  and  $P_2$  are strictly convex on  $R$ . Furthermore,  $P_3$  is strictly convex if the following function is convex

$$f = \sqrt{2^R}. \quad (53)$$

The second derivative of  $f$  is

$$f''(R) = \sqrt{2^R} \ln^2 2 / 4 > 0, \quad (54)$$

which means  $f$  is strictly convex. Therefore,  $P_3$  is also convex.

In the next, we prove that the piecewise convex function in (14) is convex. According to Appendix B, the solution space of  $(p_{ij}^U, p_{ij}^D)$  is  $C_1 \cup (\overline{C_1} \& \overline{C_2})$  or  $C_2 \cup (\overline{C_1} \& \overline{C_2})$ . We first consider  $C_1 \cup (\overline{C_1} \& \overline{C_2})$ . In this case

$$P_{ij}^{\min} = \begin{cases} P_1, & \text{if } C_1, \\ P_3, & \text{if } \overline{C_1} \& \overline{C_2}. \end{cases} \quad (55)$$

Define  $2^{R_e} = \frac{(h_j^D - h_{ij})(1 + \chi)}{h_i^U - (1 + \chi)h_{ij}}$  as the intersection of  $P_1$  and  $P_3$ . The value and the first derivative of  $P_{ij}^{\min}$  in  $R_e$  are both continuous, as

$$\begin{cases} P_1(R = R_e) = P_3(R = R_e) = (2^{R_e} - 1) / h_j^D, \\ P_1'(R_e) = P_3'(R_e) = 2^{R_e} \ln 2 / h_j^D. \end{cases} \quad (56)$$

Without loss of generality, we assume  $R_1 < R_2$  for any  $R_1, R_2 \geq 0$ . In the case that  $R_1 < R_2 \leq R_e$  or  $R_e \leq R_1 < R_2$ , we can easily derive that

$$P_{ij}^{\min}(R_2) - P_{ij}^{\min}(R_1) \geq P_{ij}^{\min}'(R_1)(R_2 - R_1), \quad (57)$$

which is a necessary and sufficient condition for the convexity of  $P_{ij}^{\min}$ .

For the case that  $R_1 < R_e < R_2$ , the left part of (57) can be decomposed as

$$\begin{cases} P_{ij}^{\min}(R_2) - P_{ij}^{\min}(R_e) \geq P_1'(R_e)(R_2 - R_e) \\ > P_1'(R_1)(R_2 - R_e), \\ P_{ij}^{\min}(R_e) - P_{ij}^{\min}(R_1) \geq P_1'(R_1)(R_e - R_1), \end{cases} \quad (58)$$

which indicates that the condition in (57) is also satisfied in this case.

Therefore, it can be concluded that  $P_{ij}^{\min}$  is convex when the solution space is  $C_1 \cup (\overline{C_1} \& \overline{C_2})$ . If the solution space is  $C_2 \cup (\overline{C_1} \& \overline{C_2})$ ,  $P_{ij}^{\min}$  can also be proved as a convex function of  $R$  with the similar approach. This ends the proof.

## APPENDIX D

Define the super-level set of  $\beta_{\text{EE}}^*$  as  $S_\alpha = \{R_{\text{tot}} | \beta_{\text{EE}}^*(R_{\text{tot}}) \geq \alpha\}$ .  $\beta_{\text{EE}}^*(R_{\text{tot}})$  is quasi-concave if  $S_\alpha$  are convex for all  $\alpha$ .

- For  $\alpha \leq 0$ ,  $S_\alpha$  is its domain, which is obviously convex.
- For  $\alpha > 0$ ,  $S_\alpha = \{R_{\text{tot}} | R_{\text{tot}} - \alpha(\omega P_{ij}^{\min}(R_{\text{tot}}) + P_e) \geq 0\}$ .

Since  $P_{ij}^{\min}(R_{\text{tot}})$  is convex,  $S_\alpha$  is convex.

Therefore, for all  $\alpha \in R$ ,  $S_\alpha$  is convex and  $\beta_{\text{EE}}^*(R_{\text{tot}})$  is quasi-concave. This ends the proof.

## APPENDIX E

Note that the constraints (20a), (20b), (20c), and (20d) are linear. Therefore, the problem in (20) is convex if the objective function is convex. In the following, we prove that the objective function,  $\sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} P_{ij}$ , is convex.

Define

$$\begin{cases} f_1(\gamma_{ij}, \hat{R}_{ij}) = \gamma_{ij} P_1(\hat{R}_{ij} / \gamma_{ij}), \\ f_2(\gamma_{ij}, \hat{R}_{ij}) = \gamma_{ij} P_2(\hat{R}_{ij} / \gamma_{ij}), \\ f_3(\gamma_{ij}, \hat{R}_{ij}) = \gamma_{ij} P_3(\hat{R}_{ij} / \gamma_{ij}), \end{cases} \quad (59)$$

where  $P_k, (k = 1, 2, 3)$  are defined in (16).

It is clear that  $f_1, f_2$ , and  $f_3$  are convex if  $G_1 = \gamma_{ij} 2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}}$  and  $G_2 = \gamma_{ij} \sqrt{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}}}$  are convex. The Hessian matrix of  $G_1$  is

$$H_{G_1} = \begin{bmatrix} \frac{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \hat{R}_{ij}^2 \ln^2 2}{\gamma_{ij}^3}, & -\frac{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \hat{R}_{ij} \ln^2 2}{\gamma_{ij}^2} \\ -\frac{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \hat{R}_{ij} \ln^2 2}{\gamma_{ij}^2}, & \frac{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \ln^2 2}{\gamma_{ij}} \end{bmatrix}, \quad (60)$$

whose eigenvalues are

$$\left(0, \frac{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \ln^2 2 (\gamma_{ij}^2 + \hat{R}_{ij}^2)}{\gamma_{ij}^3}\right)^T, \quad (61)$$

which are non-negative when  $\gamma_{ij} > 0$ . Therefore,  $G_1$  is strictly convex when  $\gamma_{ij} \in (0, 1]$ .

Similarly, the Hessian matrix of  $G_2$  is

$$H_{G_2} = \begin{bmatrix} \frac{\sqrt{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \hat{R}_{ij}^2 \ln^2 2}}{4\gamma_{ij}^3}, & -\frac{\sqrt{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \hat{R}_{ij} \ln^2 2}}{4\gamma_{ij}^2} \\ -\frac{\sqrt{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \hat{R}_{ij} \ln^2 2}}{4\gamma_{ij}^2}, & \frac{\sqrt{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \ln^2 2}}{4\gamma_{ij}} \end{bmatrix}, \quad (62)$$

whose eigenvalues are

$$\left(0, \frac{\sqrt{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \ln^2 2 (\hat{R}_{ij}^2 + \gamma_{ij}^2)}}{4\gamma_{ij}^3}\right)^T, \quad (63)$$

which are non-negative when  $\gamma_{ij} > 0$ . Therefore,  $G_2$  is strictly convex when  $\gamma_{ij} \in (0, 1]$ .

Therefore, when  $\gamma_{ij} \in (0, 1]$ ,  $f_1, f_2$ , and  $f_3$  are convex, since  $G_1$  and  $G_2$  are convex.

Now we are ready to prove that  $\sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} P_{ij}$  is convex. Denote  $f(\gamma_{ij}, \hat{R}_{ij}) = \gamma_{ij} P_{ij}(\gamma_{ij}, \hat{R}_{ij})$  in the sequel for simplicity. According to Appendix C, the solution space of  $P_{ij}$  has two cases. Therefore, the solution space of  $f$  also has two cases.

For the first case, if the solution space of  $P_{ij}$  is  $C_1 \cup (\overline{C_1} \& \overline{C_2})$ , the corresponding  $f$  can be expressed as

$$f(\gamma_{ij}, \hat{R}_{ij}) = \begin{cases} f_1(\gamma_{ij}, \hat{R}_{ij}), & \text{if } 2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \leq Z_1, \\ f_3(\gamma_{ij}, \hat{R}_{ij}), & \text{otherwise,} \end{cases} \quad (64)$$

where  $Z_1 = \frac{(h_j^D - h_{ij})(1 + \chi)}{h_i^U - (1 + \chi)h_{ij}}$ .

For the second case, if the solution space of  $P_{ij}$  is  $C_2 \cup (\overline{C_1} \& \overline{C_2})$ , the corresponding  $f$  can be expressed as

$$f(\gamma_{ij}, \hat{R}_{ij}) = \begin{cases} f_2(\gamma_{ij}, \hat{R}_{ij}), & \text{if } 2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \leq Z_2, \\ f_3(\gamma_{ij}, \hat{R}_{ij}), & \text{otherwise,} \end{cases} \quad (65)$$

where  $Z_2 = \frac{h_i^U - (1 + \chi)h_{ij}}{(h_j^D - h_{ij})(1 + \chi)}$ .

Similar to Appendix C, we first consider the first case where the solution space of  $P_{ij}$  is  $C_1 \cup (\overline{C_1} \& \overline{C_2})$ .

Define  $\mathbf{x} = (\gamma_{ij}, \hat{R}_{ij})$ ,  $A_1 = \frac{(h_j^D - h_{ij})(1 + \chi)}{h_i^U - (1 + \chi)h_{ij}}$ , and  $L = \{\mathbf{x} | 2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} = A_1\}$ . Denote  $\mathbf{x}_1 = (\gamma_1, R_1)^T$  and  $\mathbf{x}_2 = (\gamma_2, R_2)^T$ . Without loss of generality, assume  $0 \leq \frac{R_1}{\gamma_1} < \frac{R_2}{\gamma_2}$ . If  $\frac{R_1}{\gamma_1} < \frac{R_2}{\gamma_2} \leq \log_2 A_1$  or  $\log_2 A_1 \leq \frac{R_1}{\gamma_1} < \frac{R_2}{\gamma_2}$ , we can easily derive that

$$f(\mathbf{x}_2) - f(\mathbf{x}_1) \geq \nabla f(\mathbf{x}_1)^T (\mathbf{x}_2 - \mathbf{x}_1), \quad (66)$$

which is the necessary and sufficient condition for the convexity of  $f$ .

Next, we will prove that if  $\frac{R_1}{\gamma_1} < \log_2 A_1 < \frac{R_2}{\gamma_2}$ , the condition in (66) is also satisfied.

According to (56), for  $\mathbf{x} \in L$ , the first derivatives of  $f_1$  and  $f_3$  are equal, as

$$\begin{cases} \left. \frac{\partial f_1}{\partial \gamma_{ij}} \right|_{\mathbf{x} \in L} = \left. \frac{\partial f_3}{\partial \gamma_{ij}} \right|_{\mathbf{x} \in L} = \left. \frac{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} (\gamma_{ij} - \hat{R}_{ij} \ln 2) - \gamma_{ij}}{\gamma_{ij} h_j^D} \right|_{\mathbf{x} \in L}, \\ \left. \frac{\partial f_1}{\partial \hat{R}_{ij}} \right|_{\mathbf{x} \in L} = \left. \frac{\partial f_3}{\partial \hat{R}_{ij}} \right|_{\mathbf{x} \in L} = \left. \frac{2^{\frac{\hat{R}_{ij}}{\gamma_{ij}}} \ln 2}{h_j^D} \right|_{\mathbf{x} \in L}. \end{cases} \quad (67)$$

Denote  $\mathbf{x}_0 = (\gamma_2, \gamma_2 \log_2 A_1)$ . The left part of (66) can be decomposed as

$$\begin{cases} f(\mathbf{x}_2) - f(\mathbf{x}_0) \geq \nabla f_3(\mathbf{x}_0)^T (\mathbf{x}_2 - \mathbf{x}_0) \\ \quad = f_1(\mathbf{x}_0)^T (\mathbf{x}_2 - \mathbf{x}_0), \\ f(\mathbf{x}_0) - f(\mathbf{x}_1) \geq \nabla f_1(\mathbf{x}_1)^T (\mathbf{x}_0 - \mathbf{x}_1). \end{cases} \quad (68)$$

Since we have proved that  $P_1$  is convex in Appendix C, it can be derived that

$$\begin{aligned} & \nabla f_1(\mathbf{x}_0)^T (\mathbf{x}_2 - \mathbf{x}_0) - \nabla f_1(\mathbf{x}_1)^T (\mathbf{x}_2 - \mathbf{x}_0) \\ & = (P_1'(\log_2 A_1) - P_1'(\frac{R_1}{\gamma_0})) (R_2 - \gamma_2 \log_2 A_1) > 0, \end{aligned} \quad (69)$$

which indicates that

$$f(\mathbf{x}_2) - f(\mathbf{x}_0) > \nabla f_1(\mathbf{x}_1)^T (\mathbf{x}_2 - \mathbf{x}_0). \quad (70)$$

which shows that the condition in (66) is satisfied.

Hence, it can be concluded that for the first case where the solution space of  $P_{ij}$  is  $C_1 \cup (\overline{C_1} \& \overline{C_2})$ ,  $f(\gamma_{ij}, \hat{R}_{ij}) = \gamma_{ij} P_{ij}(\gamma_{ij}, \hat{R}_{ij})$  is convex. For the second case where the

solution space of  $P_{ij}$  is  $C_1 \cup (\overline{C_1} \& \overline{C_2})$ , it can also be proved that  $f(\gamma_{ij}, \hat{R}_{ij}) = \gamma_{ij} P_{ij}(\gamma_{ij}, \hat{R}_{ij})$  is convex in a similar way. Therefore, the objective function  $\sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} P_{ij}$  is convex.

## APPENDIX F

Define the super-level set of  $\beta_{EE}^*(R_{\text{tot}})$  as  $S_\theta = \{R_{\text{tot}} | \beta_{EE}^*(R_{\text{tot}}) \geq \theta\}$ , where  $\beta_{EE}^*(R_{\text{tot}}) = \frac{R_{\text{tot}}}{\omega P_{\text{tot}}^{\min} + P_{\text{fix}}} \geq \theta$  can be derived as  $R_{\text{tot}} - \theta(\omega P_{\text{tot}}^{\min} + P_{\text{fix}}) \geq 0$ . If  $S_\theta$  is convex for all  $\theta$ ,  $\beta_{EE}^*(R_{\text{tot}})$  is quasi-concave on  $R_{\text{tot}}$ .

We first prove that the super level set  $S'_\theta = \{(\{\gamma_{ij}\}_M, \{\hat{R}_{ij}\}_M) | R_{\text{tot}} - \theta(\omega P_{\text{tot}}^{\min} + P_{\text{fix}}) \geq 0\}$  is a convex set. As we have proved in Appendix E,  $P_{\text{tot}}^{\min} = \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} P_{ij}$  is a convex function of  $(\{\gamma_{ij}\}_M, \{\hat{R}_{ij}\}_M)$ . Similar to Appendix D, we can prove  $S'_\theta$  is a convex set for all  $\theta$ .

By implementing an affine transformation, where the coefficients of  $\gamma_{ij}$  are 0 and the coefficients of  $\hat{R}_{ij}$  are 1,  $(\{\gamma_{ij}\}_M, \{\hat{R}_{ij}\}_M)$  can be transformed as  $\sum_{i=1}^M \sum_{j=1}^M \hat{R}_{ij} = R_{\text{tot}}$ . Since affine transformation preserves convexity,  $S_\theta = \{R_{\text{tot}} | R_{\text{tot}} - \theta(\omega P_{\text{tot}}^{\min} + P_{\text{fix}}) \geq 0\}$  is a convex set for all  $\theta$ . Therefore,  $\beta_{EE}^*(R_{\text{tot}})$  is a quasi-concave function of  $R_{\text{tot}}$ . This ends the proof.

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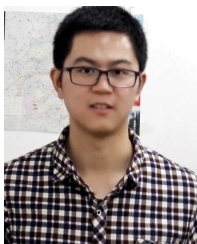
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