Optimal Base Station Sleeping in Green Cellular Networks: A Distributed Cooperative Framework Based on Game Theory

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***Abstract*—This paper proposes a distributed cooperative framework to improve the energy efficiency of green cellular networks. Based on the traffic load, neighboring base stations (BSs) cooperate to optimize the BS switching (sleeping) strategies so as to maximize the energy saving while guaranteeing users' minimal service requirements. The inter-BS cooperation is formulated following the principle of ecological self-organization. An *interaction graph* is defined to capture the network impact of the BS switching operation. Then, we formulate the problem of energy saving as a *constrained graphical game*, where each BS acts as a game player with the constraint of traffic load. The constrained graphical game is proved to be an exact constrained potential game. Furthermore, we prove the existence of a generalized Nash equilibrium (GNE), and the best GNE coincides with the optimal solution of total energy consumption minimization. Accordingly, we design a decentralized iterative algorithm to find the best GNE (i.e., the global optimum), where only local information exchange among the neighboring BSs is needed. Theoretical analysis and simulation results finally illustrate the convergence and optimality of the proposed algorithm.**

***Index Terms*—Base station sleeping, distributed cooperation, energy efficiency, potential game, generalized Nash equilibrium, decentralized algorithm, green cellular networks.**

I. INTRODUCTION

THE explosive popularity of smartphones and tablets has ignited a surging traffic demand for radio access and has been incurring massive energy consumption [1]–[3], which results in the depletion of non-renewable energy resources and causes potential harms to the environment due to CO_2 emissions (e.g., global warming) [4]. From an economic perspective, mobile cellular network operators need to spend more than 10 billion dollars on electricity to supply the energy consumption for the network, and the amount keeps growing at a rapid speed [5], [6]. Accordingly, a new research area called "green cellular networks" has recently emerged to enable various energy-efficient cellular networks [7], [8]. The operators have been seeking ways to improve energy-efficiency in all possible dimensions and all components across BSs, mobile terminals (MTs), and backhaul networks [4].

The focus of this paper is devoted to reducing the energy consumption in BSs, since BSs consume a significant portion of the whole energy used in cellular networks, reported to amount to about 60-80% [4], [9]. Recent research based on the real temporal traffic trace over one week reports that BSs are largely underutilized, while the time portion when the traffic is below 10% of peak during the day is about 30% and 45% on weekdays and weekends, respectively [8]. However, BSs with a few or even no communication activities generally consume more than 90% of their peak energy. Therefore, instead of turning off just radio transceivers, the operators probably prefer to putting the underutilized BSs into sleeping mode and transfer the imposed traffic loads to neighboring BSs during low traffic periods such as nighttime, which has substantial potential to reduce the energy squander [4].

Based on the traffic loads fluctuation, dynamically switching the operation mode of BSs to "on" or "off" is one of the effective ways to minimize the total energy consumption, which has been considered as an emerging and challenging research issue in recent years. To the best of our knowledge, [10] is the first work to study dynamic BS operations and proposed a scheme to switch off some BSs under low traffic load. Marsan *et al.* [9], [11] proposed some switching strategies for dynamic BS operations based on daily traffic profile. The problem of energy saving with BS switching is a well-known combinatorial problem, which has been proven to be NP-hard [6], [12], [13]. Moreover, solving this kind of problem generally requires a central controller as well as the global information (channel state information and traffic load information), which makes the problem more challenging. Instead of directly probing into this problem, many works [14], [15] adopted fixed switching-off patterns and then analyzed some important quality of service (QoS) metrics (e.g., the call blocking probability and the channel outage probability). In addition, some greedy algorithms have been proposed as in [4], [16]. Son *et al.* [4] designed a greedy algorithm to achieve the tradeoff between flow-level delay and energy consumption. Kim *et al.* [16] put forward a greedy algorithm to balance the energy consumption and the revenue in heterogeneous networks composed of cellular networks and wireless local area networks (WLANs).

In contrast, distributed schemes for dynamic BS switching operation [6], [17]–[20] are more favored as they do not

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require a wide-area central controller and may demand less information exchange and computational complexity [21]. Furthermore, in the newly approved 3GPP LTE specifications, a flat-system architecture is preferred instead of a traditional hierarchical structure, whereas self-organization and self-optimization capabilities are integrated to avoid failure of single point that often occurs in those centralized controllers [6], [22], [23]. Therefore, a decentralized scheme for BS operations should be designed and implemented for the future cellular communication systems. In [17], Zhou *et al.* investigated a distributed scheme implemented by mobile devices. However, the proposed distributed algorithm would cause a ping-pong effect. Then, Wong *et al.* [6] designed a decentralized algorithm implemented by both BSs and mobile devices to avoid the ping-pong effect. Additionally, Oh *et al.* [18] proposed a distributed and dynamic switching-on/off based energy saving algorithm via a newly introduced notion of network-impact. Guo *et al.* [19] took into account a distributed self-organizing-network (SON) algorithm to perform dynamic cell expansion through antenna beam tilting. However, all the existing distributed schemes were generally lacking of solid theoretic analysis on the convergence, which presents a great challenge. In [20], the authors developed a actor-critic method based transfer learning framework for BS energy saving, where the BS operations under a variant traffic load were formulated as a Markov decision processes with knowledge transferring. The convergence of the algorithm was established via ordinary differential equation and stochastic approximation theory. Nevertheless, the optimality of the obtained solution cannot be guaranteed.

In this paper, we propose a distributed cooperative framework to solve the optimal BS sleeping problem in green cellular networking scenario. The main contributions of our work are summarized as below:

- An *interaction graph* is defined to capture the network flow during BSs switching on/off. In order to deal with the conflicts and interactions among the distributed BS switching operation, we formulate the energy-saving BS sleeping problem as a *constrained graphical game*, where each BS acts as a game player under the constraint of system load. Local cooperation is employed to improve the game efficiency. Specifically, the utility function of each player is defined to consider its individual energy consumption and the energy consumption of its neighbors at the same time.
- The constrained graphical game is proved to be an exact constrained potential game. Then we prove the existence of the generalized Nash equilibrium (GNE) [32], [33], and the best GNE coincides with the optimal solution achieving the total energy consumption minimization. Furthermore, we design a decentralized iterative algorithm to find the best GNE (i.e., the global optimum), where only local information exchange is needed among the neighboring BSs. The convergence and optimality property are investigated as well.

Although local cooperation is employed in our model, each BS makes its strategy decision independently and autonomously. At each time, multiple BSs can switch their modes simultaneously. Therefore, the proposed approach is essentially distributed/decentralized. In contrast, the centralized solution means global cooperation (i.e., all the BSs cooperate with each other), which generally requires a central controller as well as the global information collected. The proposed decentralized cooperation framework incorporates the advantages of both the decentralized and centralized approaches. It needs only local information and lower computational complexity for strategy decision (advantages of decentralized approaches), but could achieve the optimal solution (advantage of centralized approaches).

The remainder of the paper is organized as follows. In Section II, we present the system model followed by the problem formulation for the energy-saving BS operations. In Section III, we formulate a constrained graphical game and investigate the properties of its GNE points. In Section IV, a decentralized iterative algorithm is proposed to find the global optimum of our problem. Section V presents simulation results and discussions. Conclusions are drawn in Section VI.

II. System Model and Problem Formulation

A. System Model

We consider a heterogeneous cellular network (a mixture of macrocells and small cells such as microcells, picocells, and femtocells). Assume that all BSs in the network operate in open access, i.e., any user is allowed to connect to the access points (namely BSs) from any tier [15]. The set of BSs, denoted by $\mathcal{K} = \{1, 2, \ldots, K\}$, lies in a two-dimensional area $\mathcal{A}$. Similar to [18], we assume the traffic arrival rate of a UE located at location x follows an independent Poisson distribution with mean arrival rate $\lambda(x)$, and its average requested file size is an exponentially distributed random variable with mean $1/\mu(x)$. Then, the traffic load of a UE located at x is defined as $\gamma(x) = \lambda(x)/\mu(x)$. By setting different arrival rates or file sizes for different users, this model can capture spatial traffic variability.

Our focus is on downlink transmissions (i.e., from BSs to user equipments (UEs)) which is a primary usage mode for the mobile Internet. A UE located at $x \in \mathcal{A}$ is associated with and served by the BS which provides the best signal strength[1] [18], i.e.,

$$k = \arg\max_{i \in \mathcal{B}_{\text{on}}} P_i^{\text{tx}} \cdot g(i, x), \tag{1}$$

where $\mathcal{B}_{\text{on}} \subseteq \mathcal{K}$ denotes the set of active BSs, P_i^{tx} is the transmission power of BS i, and $g(i, x)$ is the average channel gain from BS i to the UE at location x including the path loss and other factors such as multipath fading, log-normal shadowing. The received signal to interference and noise ratio (SINR) at location x from BS k is given by

$$\text{SINR}_k(x, \mathcal{B}_{\text{on}}) = \frac{g(k, x) P_k^{\text{tx}}}{\sum\limits_{i \in \mathcal{B}_{\text{on}} \setminus \{k\}} g(i, x) P_i^{\text{tx}} + \sigma^2}, \tag{2}$$

[1]It is noted that other user association metrics could also be used. Since the optimal user association problem has been sufficiently studied in [25]–[27], our paper simplifies the user association problem and focuses on the optimal BS sleeping scheme due to the space limitation.

TABLE I
SUMMATION OF USED NOTATIONS

Notations	Description
$\mathcal{A} \in \mathbb{R}^2$	considered geographic region
$x \in \mathcal{A}$	location in region $\mathcal{A}$
$\mathcal{K}$	set of BSs
$\mathcal{B}_{\text{on}} \subseteq \mathcal{K}$	set of active BSs
$\mathcal{A}_k$	coverage of BS k
P_k^{tx}	transmission power of BS k
P_k	maximum operational power of BS k
$q_k \in [0, 1]$	the portion of the fixed power consumption for BS k
$g(k, x)$	average channel gain from BS k to the UE at location x
$\lambda(x)$	average traffic arrival rate at location x
$1/\mu(x)$	average requested file size at location x
$\gamma(x)$	traffic load at location x
$R_k(x, \mathcal{B}_{\text{on}})$	channel capacity from BS $k \in \mathcal{B}_{\text{on}}$ to UE at location x
ρ_k	system load of BS k
ρ_k^{th}	system load threshold of BS k
$\rho_{k\to i}$	system load transferred from BS k to BS i
ϕ_k	power consumption of BS k
E	total energy expenditure in the network
G_r	interaction graph characterizing the relationship among BSs
$\mathcal{C}_k$	set of direct neighbors of BS k
$\mathcal{D}_k$	set of interacting neighbors of BS k
$\mathcal{G}$	formulated BS switching game
$\mathcal{S}_k = \{0, 1\}$	set of available switching strategy of player k
U_k	utility function of player k
f_k	correspondence function for satisfaction of the constraint
$\mathcal{S}$	joint strategy space for all the players
$\mathbf{s} \in \mathcal{S}$	strategy profile of all the players
$\mathcal{S}_{-k}$	joint strategy space of all the players excluding k
$\mathbf{s}_{-k} \in \mathcal{S}_{-k}$	strategy profile of all the players excluding k
$\mathcal{S}_{\mathcal{C}_k}$	joint strategy space for player k's direct neighbors
$\mathbf{s}_{\mathcal{C}_k} \in \mathcal{S}_{\mathcal{C}_k}$	strategy profile for player k's direct neighbors
$\mathcal{S}_{\mathcal{D}_k}$	joint strategy space for player k's interacting neighbors
$\mathbf{s}_{\mathcal{D}_k} \in \mathcal{S}_{\mathcal{D}_k}$	strategy profile for player k's interacting neighbors
Φ	potential function of the game $\mathcal{G}$
β	learning parameter of the proposed decentralized algorithm
$\pi(\mathbf{s})$	stationary distribution of any switching strategy profile $\mathbf{s}$

where σ^2 is the noise power. Then, the channel capacity from BS $k \in \mathcal{B}_{\text{on}}$ to UE at location x is computed by Shannon's formula:

$$R_k(x, \mathcal{B}_{\text{on}}) = W \log_2 (1 + \text{SINR}_k(x, \mathcal{B}_{\text{on}})), \quad (3)$$

where W denotes the channel bandwidth. Then, the system load density $\varrho_k(x, \mathcal{B}_{\text{on}})$ is defined as the fraction of time required to deliver traffic loads from BS $k \in \mathcal{B}_{\text{on}}$ to location x, namely $\varrho_k(x, \mathcal{B}_{\text{on}}) = \gamma(x)/R_k(x, \mathcal{B}_{\text{on}})$. Denoting $\mathcal{A}_k$ as BS k's coverage (i.e., the set of UEs' locations served by BS k), the system load of BS k is defined as the fraction of time to serve the total traffic load in its coverage, i.e.,

$$\rho_k(x, \mathcal{B}_{\text{on}}) = \int_{\mathcal{A}_k} \frac{\gamma(x)}{R_k(x, \mathcal{B}_{\text{on}})} dx. \quad (4)$$

For better reading, Table I summarizes the used notations in this paper.

B. Problem Formulation

1) The Cost Function of Energy: We adopt a general BS energy consumption model [4] composed of two types of power consumptions: fixed power consumption and adaptive power consumption which is proportional to BS's utilization.

$$\phi_k = \begin{cases} (1 - q_k)\rho_k P_k + q_k P_k, & k \in \mathcal{B}_{\text{on}} \\ 0, & k \notin \mathcal{B}_{\text{on}} \end{cases} \quad (5)$$

where P_k is the maximum operational power of BS k including power consumptions for transmit antennas as well as power amplifiers, cooling equipment etc., and $q_k \in [0, 1]$ is the portion of the fixed power consumption for BS k. When $q_k = 0$, BSs are assumed to consist of only energy-proportional components. Specifically, such BSs would ideally consume no power when idle, and gradually consume more power as the activity level (reflected by ρ_k) increases. This type of BSs is referred to as energy-proportional BSs, which is still far from reality because several components in the BSs dissipate standby power while even being inactive. On the other hand, the type of BSs, which consumes the fixed power in spite of its activity level unless they are totally turned off, i.e., $q_k > 0$, will be referred to as non-energy-proportional BS. In particular, when $q_k = 1$, this model can also capture a constant energy consumption model, which is widely applied in many works [4], [8]–[10], [17], [18].

2) Energy Saving Problem Formulation: In this paper, we aim at proposing a BS switching algorithm that minimizes the total energy expenditure in cellular networks. In general, our energy saving problem considering the BS switching operations can be formulated as:

$$\min_{\mathcal{B}_{\text{on}}} E(\mathcal{B}_{\text{on}}) = \sum_{k \in \mathcal{K}} \phi_k, \quad (6a)$$

$$\text{s.t.} \quad 0 \le \rho_k \le \rho_k^{\text{th}}, \forall k \in \mathcal{K}, \quad (6b)$$

$$\mathcal{B}_{\text{on}} \neq \emptyset. \quad (6c)$$

Similar to [18], we introduce a system load threshold $\rho_k^{\text{th}} \le 1$ on the system load to balance the trade-off between the energy efficiency and the system stability/reliability, as shown in the constraint of the above problem formulation (6). In specific, with a low threshold value, BSs would operate in a conservative manner with a low system load on average (i.e., large spare capacity). Consequently, users would experience less delay. Moreover, less call dropping probability can be expected since the BSs become more robust to bursty traffic arrivals. In contrast, with a high threshold value close to one (i.e., a loose threshold), we can obtain more energy saving at the cost of slight performance reduction.

Remark 1. *The energy consumption minimization problem above aims to determine the set of active BSs subject to the system load constraint, which can be proved to be NP-complete by reducing from a vertex cover problem [12], [18]. Finding an optimal solution to this problem faces the following two difficulties: First, it requires high computational complexity to find the optimal active BS set among 2^K on/off combinations, especially when the number of BSs is large; Second, it needs a centralized controller which collects the channel state information and traffic load information from all BSs in practice.*

It is worth noting that the existing works [4], [16] only propose some greedy heuristic algorithms to find a feasible solution to the problem (6), and there is no theoretic analysis on the optimality property. Moreover, the proposed algorithms require the information from all involved BSs. Then, Oh *et al.*

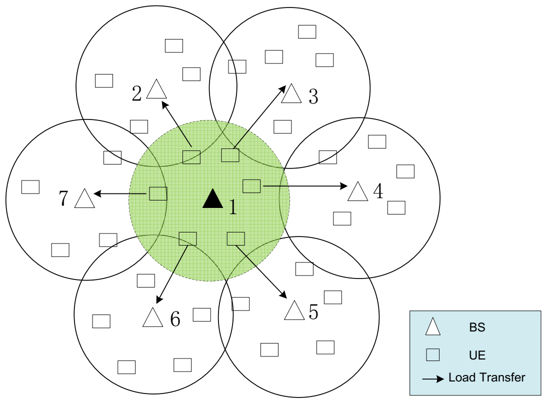

Fig. 1. BS cooperation for energy saving.

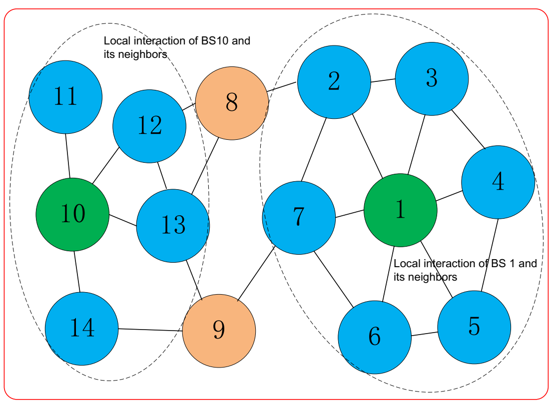

Fig. 2. Distributed network model.

[18] aims to reduce the information exchange by designing distributed algorithms. However, no optimality of the algorithm can be guaranteed.

III. Local Cooperation Game for Energy-Saving BS Switching

In this section, we discuss on the distributed optimization of the above problem (6) by using game theory [35]–[43], which is a powerful tool to analyze the interactions between distributed decision makers and thereby improve the performance of decentralized networking/framework.

A. Local Interaction and Cooperative BS Sleeping Mechanism

When one BS is turned off, apparently, this would result in an increase in the system load of neighboring BSs. This is because those UEs originally associated with the switched-off BS need to be transferred to its neighbors (as shown in Fig. 1), even they may experience lower service rates $R_k(x, \mathcal{B}_{\text{on}})$ due to farther distances between the UEs and their new serving BSs. The neighboring BSs (BSs 2-7) are serving as acceptors for BS 1 by cooperatively sharing its traffic (as shown by the arrows) and allowing BS 1 to turn into sleeping mode for saving energy. At the same time, for supporting the extended coverage zones, transmission power of these acceptors (BSs 2-7) should be adjusted.

The network model is presented in Fig. 2 to capture the local interaction among the neighboring BSs. The solid lines between any two BSs denote the interaction between them. When BS 1 and BS 10 are switched off, the local interaction domain of BS 1 and BS 10 are illustrated by the dashed lines. For instance, as described in the figure, BS 10 and its neighboring BSs 11-14 are interacting via their own local processes. Here, BS 10 may have no knowledge on the behavior of other BSs beyond its own domain (i.e., BSs 1-9). However, the neighboring BSs of BS 10 are also interacting with their respective neighboring BSs, and thus, BS 10 is directly and indirectly influencing the behavior of all the other BSs in the system. In turn, local behaviors of the BSs would generate the global behavioral pattern of the whole system. This type of interaction follows the principle of ecological self-organization [27]–[29].

Definition 1: An ***Interaction Graph*** is defined by $G_r = (\mathcal{K}, \varepsilon)$, where $\mathcal{K}$ is the set of vertices (BSs as players), ε is the set of edges. We say BSs j and k are direct neighbors if they are connected by an edge, i.e., $(j,k) \in \varepsilon$. Moreover, we define $\mathcal{C}_k$ as the set of BS k's neighbors, $\mathcal{C}_k = \{j : (j,k) \in \varepsilon\}$. Notably, $j \in \mathcal{C}_k \Leftrightarrow k \in \mathcal{C}_j$.

B. Graphical Game Model

Then, the graphical game model is formally denoted by $\mathcal{G} = \left[G_r, \{\mathcal{S}_k\}_{k\in\mathcal{K}}, \{U_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}}\right]$, where $\mathcal{S}_k = \{0, 1\}$ (0 denotes "off", 1 denotes "on") is the set of available switching strategy for player (BS) k, U_k is the utility function of player k to evaluate the energy consumption, and f_k represents a correspondence function for satisfaction of the constraint [32]. An strategy profile of all the players is a vector, denoted by $\mathbf{s} = (s_1, s_2, \ldots, s_K) \in \mathcal{S}$, where $\mathcal{S} = \mathcal{S}_1 \otimes \mathcal{S}_2 \otimes \cdots \otimes \mathcal{S}_K$ represents the joint strategy space for all the players. Besides, the strategy profile of all the players excluding k is denoted by $\mathbf{s}_{-k} = (s_1, \ldots, s_{k-1}, s_{k+1} \ldots, s_K) \in \mathcal{S}_{-k}$, where $\mathcal{S}_{-k} = \mathcal{S}_1 \otimes \cdots \otimes \mathcal{S}_{k-1} \otimes \mathcal{S}_{k+1} \otimes \cdots \otimes \mathcal{S}_K$. Additionally, the strategy profile of player k's neighbors is denoted by $\mathbf{s}_{\mathcal{C}_k} \in \mathcal{S}_{\mathcal{C}_k}$, where $\mathcal{S}_{\mathcal{C}_k} = \otimes_{j\in\mathcal{C}_k} \mathcal{S}_j$ represents the joint strategy space for player k's direct neighbors.

Based on the interaction graph, the energy consumption of each player (say player k) only depends on the strategies of itself and its neighbors, and thus can be denoted by $\phi_k(s_k, \mathbf{s}_{\mathcal{C}_k})$. Moreover, because 1) the neighboring BSs connected though high-speed wireline can easily communicate with each other (e.g., over X2 interface in LTE), 2) efficiency of the game can be greatly improved by local cooperation, we design the utility function as

$$U_k(s_k, \mathbf{s}_{\mathcal{D}_k}) = -\left(\phi_k(s_k, \mathbf{s}_{\mathcal{C}_k}) + \sum_{i\in\mathcal{C}_k} \phi_i(s_i, \mathbf{s}_{\mathcal{C}_i})\right), \quad (7)$$

where $\mathcal{D}_k = \mathcal{C}_k \bigcup_{i\in\mathcal{C}_k} \mathcal{C}_i$ includes player k's direct neighbors $\mathcal{C}_k$ as well as indirect neighbors $\bigcup_{i\in\mathcal{C}_k} \mathcal{C}_i$. Accordingly, $\mathbf{s}_{\mathcal{D}_k}$ denotes the strategy profile of player k's interacting neighbors[2], and $\mathcal{S}_{\mathcal{D}_k}$ represents their joint strategy space. It is worth

[2]To avoid confusion, we refer to the direct and indirect neighbors $\mathcal{D}_k$ as interacting neighbors in the remainder of this paper.

noting that the above defined utility function is comprised of two parts: the individual energy consumption of player k and the aggregate energy consumption of its direct neighbors. In other words, when a player makes a decision, it should not only consider itself but also considers its direct neighbors.

In our game model, $f_k : \mathcal{S}_{\mathcal{D}_k} \to \aleph_k$ is a correspondence which determines the set of satisfied actions of player k given the actions played by player k's interacting neighbors. According to the local interaction rule, the system load of player k can be denoted by $\rho_k\left(s_k, \mathbf{s}_{\mathcal{C}_k}\right)$, and player k's switching strategy can affect the system load of itself and its neighbors. In order to guarantee the system load below the threshold ρ_i^{th}, we define the correspondence by $f_k\left(\mathbf{s}_{\mathcal{D}_k}\right) = \left\{s_k \in \mathcal{S}_k : 0 \leq \rho_i\left(s_i, \mathbf{s}_{\mathcal{C}_i}\right) \leq \rho_i^{\text{th}}, \forall i \in \mathcal{C}_k \cup \{k\}\right\}$. Obviously, $\aleph_k = f_k\left(\mathbf{s}_{\mathcal{D}_k}\right)$ is a subset of the set $\mathcal{S}_k$, i.e., $\aleph_k \subseteq \mathcal{S}_k$.

Then, the BS operation game with local cooperation is expressed as

$$(\mathcal{G}) : \max_{s_k \in f_k\left(\mathbf{s}_{\mathcal{D}_k}\right)} U_k\left(s_k, \mathbf{s}_{\mathcal{D}_k}\right), \forall k \in \mathcal{K}. \tag{8}$$

The GNE under pure strategy in games of normal form with constrained set of actions, as introduced by Debreu in [30] and later by Rosen in [31], can be defined as follows.

*Definition 2 (**Generalized Nash equilibrium (GNE)**):* A BS operation profile $\mathbf{s}^* = \left(s_1^*, s_2^*, \ldots, s_K^*\right) \in \mathcal{S}$ is a pure strategy GNE of the game $\mathcal{G}$ if and only if

$$\forall k \in \mathcal{K}, s_k^* \in f_k\left(\mathbf{s}_{\mathcal{D}_k}^*\right), \tag{9a}$$

$$\forall s_k \in f_k\left(\mathbf{s}_{\mathcal{D}_k}^*\right), U_k\left(s_k^*, \mathbf{s}_{\mathcal{D}_k}^*\right) \geq U_k\left(s_k, \mathbf{s}_{\mathcal{D}_k}^*\right). \tag{9b}$$

Let the set $\mathcal{F}_k \subseteq \mathcal{S}$ be the graph of the correspondence f_k, i.e., $\mathcal{F}_k = \left\{\left(s_k, \mathbf{s}_{-k}\right) : s_k \in f_k\left(\mathbf{s}_{\mathcal{D}_k}\right)\right\}$. The set $\mathcal{F}_k$ determines the action profiles which can be observed as outcomes of the game $\mathcal{G}$, when only player k is allowed to play actions belonging to the set $f_k\left(\mathbf{s}_{\mathcal{D}_k}\right)$ given any action profile $\mathbf{s}_{\mathcal{D}_k}$. Then, $\hat{\mathcal{F}} = \bigcup_{k\in\mathcal{K}} \mathcal{F}_k$ contains all possible unilateral deviations, while $\mathcal{F} = \bigcap_{k\in\mathcal{K}} \mathcal{F}_k$ corresponds to the set of all possible (feasible) outcomes of the game $\mathcal{G}$. Notably, the definition of $\mathcal{F}$ coincides with the constraint (9a) for the GNE.

*Definition 3 (**Exact Constrained Potential Game (ECPG)**):* Any game of normal form with constrained set of actions $\mathcal{G} = \left[G_r, \{\mathcal{S}_k\}_{k\in\mathcal{K}}, \{U_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}}\right]$ is an ECPG if there exists a function $\Phi : \hat{\mathcal{F}} \to \mathbb{R}$ such that for all $\mathbf{s} \in \hat{\mathcal{F}}$, it holds that, $\forall k \in \mathcal{K}$, and $\forall s_k' \in f_k\left(\mathbf{s}_{\mathcal{D}_k}\right)$,

$$U_k(s_k', \mathbf{s}_{\mathcal{D}_k}) - U_k(s_k, \mathbf{s}_{\mathcal{D}_k}) = \Phi(s_k', \mathbf{s}_{-k}) - \Phi(s_k, \mathbf{s}_{-k}). \tag{10}$$

The function Φ is called an exact potential function for the constrained game $\mathcal{G}$.

C. Analysis of GNE

Theorem 1. *The BS operation game $\mathcal{G}$ is an exact constrained potential game.*

Proof: The following steps are inspired by the similar proof given in [36], [37].

First we construct a potential function as

$$\Phi\left(s_k, \mathbf{s}_{-k}\right) = -\sum_{k\in\mathcal{K}} \phi_k\left(s_k, \mathbf{s}_{-k}\right), \tag{11}$$

which is equal to the negative of the total BS energy consumption E as defined in Eq. (6a). Since $\phi_k\left(s_k, \mathbf{s}_{-k}\right) = \phi_k\left(s_k, \mathbf{s}_{\mathcal{C}_k}\right)$ due to the local interaction rule, we have

$$\Phi\left(s_k, \mathbf{s}_{-k}\right) = -\sum_{k\in\mathcal{K}} \phi_k\left(s_k, \mathbf{s}_{\mathcal{C}_k}\right). \tag{12}$$

Suppose that an arbitrary player, say k, unilaterally changes its strategy from s_k to $s_k' \in f_k\left(\mathbf{s}_{\mathcal{D}_k}\right)$, then the change of the potential function caused by this unilateral change is given by

$$\begin{aligned} &\Phi\left(s_k', \mathbf{s}_{-k}\right) - \Phi\left(s_k, \mathbf{s}_{-k}\right) \\ &= \phi_k(s_k, \mathbf{s}_{\mathcal{C}_k}) - \phi_k(s_k', \mathbf{s}_{\mathcal{C}_k}) + \sum_{i\in\mathcal{C}_k} \left(\phi_i(s_i, \mathbf{s}_{\mathcal{C}_i}) - \phi_i(s_i, \mathbf{s}'_{\mathcal{C}_i})\right) \\ &\quad + \sum_{i\in\mathcal{K}\setminus\{\mathcal{C}_k\cup\{k\}\}} \left(\phi_i\left(s_i, \mathbf{s}_{\mathcal{C}_i}\right) - \phi_i\left(s_i, \mathbf{s}'_{\mathcal{C}_i}\right)\right), \end{aligned} \tag{13}$$

where $\phi_i\left(s_i, \mathbf{s}'_{\mathcal{C}_i}\right)$ denotes the consumed energy of player i after player k unilaterally changing the action. Since player k's action only affects the energy consumption of itself and its neighbors, we have:

$$\phi_i\left(s_i, \mathbf{s}'_{\mathcal{C}_i}\right) = \phi_i\left(s_i, \mathbf{s}_{\mathcal{C}_i}\right), \quad \forall i \in \mathcal{K} \setminus \{\mathcal{C}_k \cup \{k\}\}. \tag{14}$$

On the other hand, the change of individual utility function caused by this unilaterally change is given by

$$\begin{aligned} &U_k\left(s_k', \mathbf{s}_{\mathcal{D}_k}\right) - U_k\left(s_k, \mathbf{s}_{\mathcal{D}_k}\right) \\ &= \phi_k(s_k, \mathbf{s}_{\mathcal{C}_k}) - \phi_k(s_k', \mathbf{s}_{\mathcal{C}_k}) + \sum_{i\in\mathcal{C}_k} \left(\phi_i(s_i, \mathbf{s}_{\mathcal{C}_i}) - \phi_i(s_i, \mathbf{s}'_{\mathcal{C}_i})\right). \end{aligned} \tag{15}$$

Then, according to (13)-(15), we can get

$$U_k(s_k', \mathbf{s}_{\mathcal{D}_k}) - U_k(s_k, \mathbf{s}_{\mathcal{D}_k}) = \Phi(s_k', \mathbf{s}_{-k}) - \Phi(s_k, \mathbf{s}_{-k}). \tag{16}$$

That is, the change in individual utility function caused by any player's unilateral deviation is equal to the change in the potential function. Thus, according to the definition, $\mathcal{G}$ is an exact constrained potential game. This concludes the proof. ■

Before we continue, it is necessary to state that not all the properties of potential games [44] hold for the constrained potential games. For instance, not all exact constrained potential games have an equilibrium, since unilateral deviations of a set of players from any action profile $\mathbf{s} \in \mathcal{F}$ might lead to action profiles which do not belong to $\mathcal{F}$ (refer to [32] for a comprehensive review on this problem). In the following, we analyze the existence and optimality of the equilibrium in our BS operation game.

Theorem 2. *The exact constrained potential game $\mathcal{G} = \left[G_r, \{\mathcal{S}_k\}_{k\in\mathcal{K}}, \{U_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}}\right]$ with potential function $\Phi : \hat{\mathcal{F}} \to \mathbb{R}$, has at least one GNE in pure strategy, if the traffic is under-loaded when the BSs are all on, i.e., $0 \leq \rho_k \leq \rho^{th}$, $\forall k \in \mathcal{K}$.*

Proof: By assumption, $\mathbf{s}^0 = \left(s_k, \mathbf{s}_{-k}\right) = (1, 1, \ldots, 1)$ is a feasible solution, i.e., $\mathbf{s}^0 \in \mathcal{F}$. Thus, there exists at least one feasible outcome for the game.

Now, $\forall k \in \mathcal{K}$, any unilateral deviation of player k from $\mathbf{s}^0$ leads to an action profile $\mathbf{s}' = \left(s_k', \mathbf{s}_{-k}\right) \in \mathcal{F}_k$, i.e., $s_k' \in f_k\left(\mathbf{s}_{\mathcal{D}_k}\right)$. Because player k's switching strategy can only affect the system load of itself and its neighbors, and the correspondence function f_k is defined to guarantee that the system load of player k and its neighbors satisfy the constraint, the unilateral deviation from a feasible action profile $\mathbf{s}^0$ is also a feasible action profile. Thus, $\mathbf{s}^1 = (s'_k, \mathbf{s}_{-k}) \in \mathcal{F}$. Moreover, since $s'_k \in f_k(\mathbf{s}_{\mathcal{D}_k})$, according to Theorem 1, we have

$$U_k(s'_k, \mathbf{s}_{\mathcal{D}_k}) - U_k(s_k, \mathbf{s}_{\mathcal{D}_k}) = \Phi(s'_k, \mathbf{s}_{-k}) - \Phi(s_k, \mathbf{s}_{-k}). \tag{17}$$

Therefore, $U_k(s'_k, \mathbf{s}_{\mathcal{D}_k}) > U_k(s_k, \mathbf{s}_{\mathcal{D}_k})$ results in $\Phi(s'_k, \mathbf{s}_{-k}) > \Phi(s_k, \mathbf{s}_{-k})$. Furthermore, due to the nature of game, player k unilaterally deviates its strategy from the original strategy s_k to s'_k only when $U_k(s'_k, \mathbf{s}_{\mathcal{D}_k}) > U_k(s_k, \mathbf{s}_{\mathcal{D}_k})$. Hence, we have $\Phi(s'_k, \mathbf{s}_{-k}) > \Phi(s_k, \mathbf{s}_{-k})$, i.e., $\Phi(\mathbf{s}^1) > \Phi(\mathbf{s}^0)$. In this way, unilateral deviations of all the players would achieve such a feasible improvement path $\{\mathbf{s}^0, \mathbf{s}^1, \mathbf{s}^2, \ldots\} \subseteq \mathcal{F}$, which conforms to $\Phi(\mathbf{s}^0) < \Phi(\mathbf{s}^1) < \Phi(\mathbf{s}^2) < \cdots$.

Besides, because the number of feasible strategy profiles is finite ($|\mathcal{F}| \leq 2^{|\mathcal{K}|}$), the above improvement path must be finite and terminate in one pure strategy GNE point $\mathbf{s}^*$, where no player could unilaterally deviate to increase its utility (decrease its energy consumption), still keeping the system load constraint satisfied. Moreover, each GNE maximizes the potential function Φ in the feasible region, either locally or globally [44]. ■

Remark 2. *We assume in the proof of Theorem 2 that the traffic is under-loaded when the BSs are all on, i.e.,* $0 \leq \rho_k \leq \rho_k^{th}$, $\forall k \in \mathcal{K}$. *This is a very reasonable assumption, since the motivation for us to perform energy-saving BS sleeping is based on the fact that BSs are largely underutilized. Therefore, the assumption always holds.*

Theorem 3. *For the formulated exact constrained potential games* $\mathcal{G}$, *each GNE locally or globally minimizes the total BS energy consumption* E *under the system load constraint, and the best GNE is the global optimum for minimizing the total BS energy consumption* E.

Proof: According to the definition of GNE, for an arbitrary GNE $\mathbf{s}^*$, we have $s_k^* \in f_k(\mathbf{s}^*_{\mathcal{D}_k}), \forall k \in \mathcal{K}$. Then, based on the definition of correspondence f_k, we know that all the GNE are in the feasible region, i.e., $0 \leq \rho_k \leq \rho_k^{\text{th}}$, $\forall k \in \mathcal{K}$.

Besides, we have proved in Theorem 2 that all GNE are the maximizers of the potential function Φ in the feasible region, either locally or globally. Furthermore, according to Eq. (6a) and Eq. (11), the potential function Φ is exactly equal to the negative of the total BS energy consumption E. That is, maximizing the potential function Φ is equivalent to minimizing the total BS energy consumption E. Therefore, each GNE of the game $\mathcal{G}$ locally or globally minimizes the total BS energy consumption E under the system load constraint, and the best GNE is the global optimum for minimizing the total BS energy consumption E. ■

IV. Decentralized Iterative Algorithm for Achieving Globally Optimal Solution

As we have proved in section III, the distributed implementation by our formulated game model can obtain very attractive equilibrium solutions that minimize the network energy consumption. In this section, we propose a decentralized iterative algorithm to find the optimal solution to the BS sleeping problem in (6). According to Theorem 3, in order to achieve the global optimum, we only need to develop an effective algorithm to obtain the best GNE.

A. Algorithm Description

Let $s_k(t)$ be the switching strategy of BS k at iteration t, respectively, for $k = 1, 2, \ldots, K$ and $t \geq 0$. The proposed procedure is described in Algorithm 1 and the schematic diagram is shown in Fig. 3.

Algorithm 1: Decentralized BS Sleeping Algorithm

Initialization: At the initial time $t = 0$, let each BS on operating, i.e., $s_k(0) = 1$, $\forall k \in \mathcal{K}$.

Loop for $t = 0, 1, 2, \ldots$

1) **Player selection:** A set of non-interacting BSs, $\mathcal{M}(t)$, is randomly selected in an distributed and autonomous manner[3]. Then, the selected BSs calculate their current utility value $U_k(t)$ by using Eq. (7) through necessary communication with neighboring BSs.
2) **Switching Strategy Exploration:** Each BS $k \in \mathcal{M}(t)$ independently and autonomously explores a new switching strategy $\hat{s}_k(t) \in \{0, 1\}$, and $\hat{s}_k(t) \neq s_k(t)$. If $\hat{s}_k(t) = 0$ (to be switched off), check whether $\hat{s}_k(t)$ satisfies the following feasibility constraint:

$$\underbrace{\int_{\mathcal{A}_i} \frac{\gamma(x)}{R_i(x)} dx}_{\rho_i} + \underbrace{\int_{\mathcal{A}_{k\to i}} \frac{\gamma(x)}{R_i(x)} dx}_{\rho_{k\to i}} \leq \rho_i^{\text{th}}, \forall i \in \mathcal{C}_k \cap \mathcal{B}_{\text{on}}, \tag{18}$$

where $\mathcal{A}_{k\to i}$ is the coverage of UEs who will be handed over from BS k to the neighboring (on-operation) BS i when BS k is switched off. ρ_i is the original system load defined as the internal system load of BS i, and the external system load increment $\rho_{k\to i}$ is the system load transferred from BS k to BS i due to the neighboring BS's switching off. Accordingly, if $\hat{s}_k(t) = 1$ (to be switched on), we should check the following feasibility constraint:

$$\sum_{i \in \mathcal{C}_k \cap \mathcal{B}_{\text{on}}} \underbrace{\int_{\mathcal{A}_{i\to k}} \frac{\gamma(x)}{R_k(x)} dx}_{\rho_{i\to k}} \leq \rho_k^{\text{th}}. \tag{19}$$

If the above constraints hold[4], BS k adheres to its selection $\hat{s}_k(t)$ in an estimation period and then its neighbors (including BS k) calculate their respective energy consumption according to Eq. (5). Then, the selected BS $k \in \mathcal{M}(t)$ computes its explored utility value $\hat{U}_k(t)$ by Eq. (7) through the communication with its neighbors.

[3]The selection of the non-interacting BSs set can be implemented through contention mechanisms over a common control channel or a priority-based method in [34], which is omitted here for brevity.

[4]If the constraints does not hold, it means BS k cannot be switched on/off. Therefore, BS k will not perform any operation.

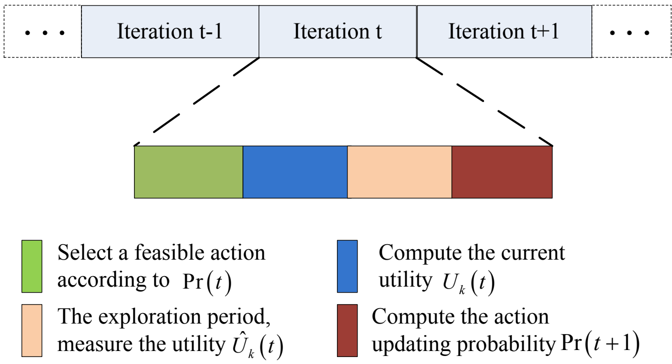

Fig. 3. The schematic diagram of the proposed decentralized iterative algorithm.

3) **Switching Strategy Updating:** BS $k \in \mathcal{M}(t)$ updates its switching strategy according to the following rule:

$$\begin{cases} \Pr\left(s_k\left(t+1\right) = \hat{s}_k\left(t\right)\right) = \dfrac{\exp\left\{\beta \hat{U}_k\left(t\right)\right\}}{\Psi} \\ \Pr\left(s_k\left(t+1\right) = s_k\left(t\right)\right) = \dfrac{\exp\left\{\beta U_k\left(t\right)\right\}}{\Psi}, \end{cases} \tag{20}$$

where $\Pr(\cdot)$ denotes the probability of the event in $(\cdot)$, $\Psi = \exp\left\{\beta U_k\left(t\right)\right\} + \exp\left\{\beta \hat{U}_k\left(t\right)\right\}$, and β is a learning parameter. Meanwhile, all other BSs keep their switching strategies unchanged, i.e., $s_i\left(t+1\right) = s_i\left(t\right), \forall i \in \mathcal{K} \backslash \mathcal{M}\left(t\right)$. Then, the UEs originally associated with the switched-off BSs are transferred to the neighbors according to the received radio signal strength.

End loop until the maximal number of iterations is reached.

The proposed decentralized iterative algorithm is inspired by the work of probabilistic decision making in [45], and lately this specific work has been further developed and applied to wireless mesh networks [46], [47]. In step 3 of the proposed algorithm, the probability of strategy updating is given by the Boltzmann distribution [48], [49], and the parameter β is analogous to the inverse of temperature in simulated annealing. When a BS explores a strategy better than its current strategy, the Boltzmann distribution will assign a higher probability to update its switching strategy; otherwise, the BS is more willing to remain the current strategy. We introduce such a probabilistic strategy updating into our energy-saving BS operation problem in order to escape the trap from sub-optimal GNE (local minima) and finally converge to the best GNE (globally optimal solution).

In the proposed algorithm, the interacting neighbors are not allowed to simultaneously change their switching strategies in order to avoid the ping-pong effect, as shown in step 1. Moreover, BSs do not really switch their modes in each iteration until the algorithm converges so as to reduce the switching cost. Each switched-off BS transfers its UEs to their neighbors, which cooperatively share its traffic. Since neighboring BSs are connected though high-speed wireline, communication among them is very easy. Neighboring BSs are cooperated by exchanging information directly and locally. By exchanging necessary information with neighboring BSs,

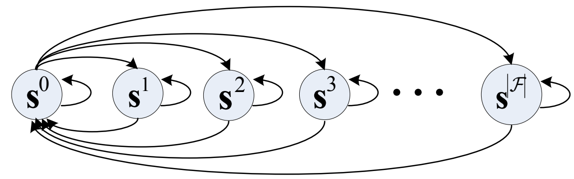

Fig. 4. Markov chain describing the proposed decentralized iterative algorithm.

each BS knows the average channel gain $g(k,x)$ from all BSs to each UE, and the transmission power of each BS P_k^{tx}. Thus, each BS can compute the updated service rate R_k as Eq. (3). Besides, the mean arrival rate $\lambda(x)$ and the average requested file size $1/\mu(x)$ can be easily obtained by each BS. Thus, each BS can calculate its system load by Eq. (4). Also, the energy consumption of each BS can be obtained by Eq. (5).

B. Stability and Optimality Analysis

Theorem 4. *If all the BSs adhere to the proposed decentralized iterative algorithm, the unique stationary distribution $\pi(\mathbf{s})$ of any switching strategy profile $\mathbf{s}$, is given by:*

$$\pi\left(\mathbf{s}\right) = \frac{\exp\left\{\beta \Phi\left(\mathbf{s}\right)\right\}}{\sum_{\hat{\mathbf{s}} \in \mathcal{S}} \exp\left\{\beta \Phi\left(\hat{\mathbf{s}}\right)\right\}}, \tag{21}$$

where $\mathcal{S}$ is the space of switching strategy profile for all the BSs, Φ is the potential function given in Eq. (11).

Proof: Denote all the BSs' on/off state vector at the t-th iteration as $\mathbf{s}\left(t\right) = \left(s_1\left(t\right), s_2\left(t\right), \ldots, s_K\left(t\right)\right)$. Notably, $\mathbf{s}\left(t\right)$ is a discrete time Markov process, as shown in Fig. 4. All the feasible solutions $\left\{\mathbf{s}^0, \mathbf{s}^1, \ldots, \mathbf{s}^{|\mathcal{F}|}\right\}$ constitute the states of the Markov chain. The initial state $\mathbf{s}^0$ (i.e., all BSs on-operation) communicates with any other state in the Markov chain. Thus, any two states communicate with each other. Moreover, according to Eq. (20), any state possesses a positive probability to keep unmoved. Therefore, the Markov chain is irreducible and aperiodic [52]. Accordingly, it has a unique stationary distribution which satisfies the following balanced equation:

$$\sum\nolimits_{X \in \mathcal{S}} \pi\left(X\right) \Pr\left(Y|X\right) = \pi\left(Y\right), \tag{22}$$

where $X, Y \in \mathcal{S}$ denote any two arbitrary network states, and $\Pr\left(Y|X\right)$ is the transition probability from X to Y.

Similar to [37], [45], [51], in the following part, we will show that the stationary distribution given by Eq. (21) satisfies the above balanced equation. For the convenience of analysis, we denote X by $\left(s_1, s_2, \ldots, s_K\right)$, where the iteration index t is omitted. In the proposed algorithm, only non-interacting BSs are allowed to update their strategies simultaneously at each iteration, which leads to the change of corresponding elements in X. With no loss of generality, we assume that the set of non-interacting BSs which simultaneously update their strategies is $\mathcal{M} = \left\{1, 2, \ldots, |\mathcal{M}|\right\}$, where $|\mathcal{M}|$ represents the number of $\mathcal{M}$'s elements. Therefore, the network state X transfers to $Y = \left(s_1', s_2', \ldots, s_{|\mathcal{M}|}', s_{|\mathcal{M}|+1}, s_{|\mathcal{M}|+2}, \ldots, s_K\right)$. We assume the probability of $\mathcal{M}$ chosen as the set of updating players is θ, and then we can get the following equation:

$$\pi(X)\Pr(Y|X) = \frac{\exp\{\beta\Phi(X)\}}{\sum_{\hat{\mathbf{s}}\in\mathcal{S}}\exp\{\beta\Phi(\hat{\mathbf{s}})\}} \times \theta \times \prod_{i\in\mathcal{M}} \frac{\exp\{\beta U_i(s_i', \mathbf{s}_{\mathcal{D}_i})\}}{\exp\{\beta U_i(s_i, \mathbf{s}_{\mathcal{D}_i})\} + \exp\{\beta U_i(s_i', \mathbf{s}_{\mathcal{D}_i})\}}. \tag{23}$$

Denoting α as

$$\alpha = \frac{\theta}{\sum_{\hat{\mathbf{s}}\in\mathcal{S}}\exp\{\beta\Phi(\hat{\mathbf{s}})\}} \times \prod_{i\in\mathcal{M}} \frac{1}{\exp\{\beta U_i(s_i, \mathbf{s}_{\mathcal{D}_i})\} + \exp\{\beta U_i(s_i', \mathbf{s}_{\mathcal{D}_i})\}}, \tag{24}$$

we obtain

$$\begin{aligned} &\pi(X)\Pr(Y|X) \\ &= \alpha \exp\{\beta\Phi(X)\} \prod_{i\in\mathcal{M}} \exp\{\beta U_i(s_i', \mathbf{s}_{\mathcal{D}_i})\} \\ &= \alpha \exp\left\{\beta\Phi(X) + \beta\sum_{i\in\mathcal{M}} U_i(s_i', \mathbf{s}_{\mathcal{D}_i})\right\}. \end{aligned} \tag{25}$$

According to the symmetry property, we can also derive

$$\pi(Y)\Pr(X|Y) = \alpha \exp\left\{\beta\Phi(Y) + \beta\sum_{i\in\mathcal{M}} U_i(s_i, \mathbf{s}_{\mathcal{D}_i})\right\}. \tag{26}$$

Construct a sequence as $X_0, X_1, X_2, \ldots, X_{|\mathcal{M}|}$, where $X_0 = X$ and $X_i = (s_1', s_2', \ldots, s_i', s_{i+1}, s_{i+2}, \ldots, s_K)$, $\forall i \in \mathcal{M}$. Notably, $Y = X_{|\mathcal{M}|}$. Then, we have

$$\begin{aligned} &\Phi(Y) - \Phi(X) = \Phi\left(X_{|\mathcal{M}|}\right) - \Phi(X_0) \\ &= \sum_{i\in\mathcal{M}} (\Phi(X_i) - \Phi(X_{i-1})) = \sum_{i\in\mathcal{M}} (U_i(X_i) - U_i(X_{i-1})). \end{aligned} \tag{27}$$

Besides, BSs in $\mathcal{M}$ are not mutually interacting neighbors, i.e., $\forall i, j \in \mathcal{M}$, $i \notin \mathcal{D}_j$. Thus, the following equation holds:

$$U_i(X_i) - U_i(X_{i-1}) = U_i(s_i', \mathbf{s}_{\mathcal{D}_i}) - U_i(s_i, \mathbf{s}_{\mathcal{D}_i}). \tag{28}$$

Then, according to Eqs. (27) and (28), we can get

$$\Phi(Y) - \Phi(X) = \sum_{i\in\mathcal{M}} (U_i(s_i', \mathbf{s}_{\mathcal{D}_i}) - U_i(s_i, \mathbf{s}_{\mathcal{D}_i})). \tag{29}$$

Now, applying Eqs. (25) and (26), we can derive the following balanced equation:

$$\pi(X)\Pr(Y|X) = \pi(Y)\Pr(X|Y). \tag{30}$$

Therefore, we have

$$\begin{aligned} &\sum_{X\in\mathcal{S}} \pi(X)\Pr(Y|X) = \sum_{X\in\mathcal{S}} \pi(Y)\Pr(X|Y) \\ &= \pi(Y) \sum_{X\in\mathcal{S}} \Pr(X|Y) = \pi(Y), \end{aligned} \tag{31}$$

which means the distribution given by Eq. (21) satisfies the balanced stationary equation (22) of the Markov process $\mathbf{s}(t)$. Moreover, the stationary distribution has been proved to be unique, thus its stationary distribution must be Eq. (21). This concludes the proof. ∎

Theorem 5. *With a sufficiently large β, the proposed algorithm achieves the globally optimal solution of the total energy consumption minimization problem with arbitrarily high probability.*

Proof: According to Theorem 3, the global optimum is the best pure strategy GNE of the constrained potential game $\mathcal{G}$, which globally maximizes the potential function. Let $\mathbf{s}^{\text{opt}}$ denote the (unique) globally optimal BS operation strategy, and thus we have

$$\mathbf{s}^{\text{opt}} = \arg\max_{\mathbf{s}\in\mathcal{S}} \Phi(\mathbf{s}). \tag{32}$$

Hence, $\forall \mathbf{s} \in \mathcal{S}\backslash\{\mathbf{s}^{\text{opt}}\}$, $\Phi(\mathbf{s}) < \Phi(\mathbf{s}^{\text{opt}})$. In addition, Theorem 4 shows that the proposed algorithm converges to a unique stationary distribution $\pi(\mathbf{s})$ given by Eq. (21), which depends on the parameter β. When β is sufficiently large, $\exp\{\beta\Phi(\mathbf{s})\} \ll \exp\{\beta\Phi(\mathbf{s}^{\text{opt}})\}$. Therefore, according to Eq. (21), we can derive

$$\lim_{\beta\to\infty} \pi\left(\mathbf{s}^{\text{opt}}\right) = 1, \tag{33}$$

and

$$\lim_{\beta\to\infty} \pi(\mathbf{s}) = 0, \forall \mathbf{s} \in \mathcal{S}\backslash\left\{\mathbf{s}^{\text{opt}}\right\}. \tag{34}$$

That is, the globally optimal solution $\mathbf{s}^{\text{opt}}$ is in probability 1, while other solutions are in probability 0. Therefore, we can concludes that the proposed algorithm can achieve the global optimum with arbitrarily high probability. ∎

Remark 3. *The discussed optimal BS sleeping solution is based on the association metric of strongest received signal strength given by Eq. (1). However, it is worth noting that no matter which association metric is employed, the corresponding optimal BS sleeping solution can be achieved by using the proposed scheme, which can be strictly proved by following the previous lines. Therefore, the terminology "optimal BS sleeping solution" means it is optimal for any given specific association metric.*

C. Convergence Speed

In this part, we will analyze how fast the algorithm converges to the global optimum, and how the parameters affect the converge behavior. For analytical convenience, we denote the set of global optima by $\mathcal{S}^{\text{opt}}$ and let $\omega_t = \Pr(\mathbf{s}(t) \in \mathcal{S}^{\text{opt}})$, $t = 0, 1, 2, \ldots$. Besides, define D as the one-step transition diameter which is the smallest integer so that, starting from any state $\mathbf{s}(0) \in \mathcal{S}$, the algorithm can reach a state in $\mathcal{S}^{\text{opt}}$ in no more than D steps following a sequence of states $\{\mathbf{s}(0), \mathbf{s}(1), \mathbf{s}(2), \ldots\}$, wherein the strategy of only one BS is altered in each transition. Following the similar lines of proof given in [34], we can obtain the following results.

Theorem 6. *The number of iterations for reaching the global optimum for the first time is upper bounded by*

$$\frac{D(1-\omega_0)}{\xi^D}, \tag{35}$$

where $\xi = \min_{\beta}\left(\frac{1}{1+\exp\left\{\beta\left|\hat{U}_k(t) - U_k(t)\right|\right\}}\right)$.

$$\begin{aligned}\omega_t &= \Pr\left(\tilde{\mathbf{s}}(t) \in \mathcal{S}^{\mathrm{opt}} \,\middle|\, \tilde{\mathbf{s}}(t-D) \in \mathcal{S}^{\mathrm{opt}}\right) \Pr\left(\tilde{\mathbf{s}}(t-D) \in \mathcal{S}^{\mathrm{opt}}\right) \\ &\quad + \Pr\left(\tilde{\mathbf{s}}(t) \in \mathcal{S}^{\mathrm{opt}} \,\middle|\, \tilde{\mathbf{s}}(t-D) \in \mathcal{S}\backslash\mathcal{S}^{\mathrm{opt}}\right) \Pr\left(\tilde{\mathbf{s}}(t-D) \in \mathcal{S}\backslash\mathcal{S}^{\mathrm{opt}}\right) \\ &= \Pr\left(\tilde{\mathbf{s}}(t-D) \in \mathcal{S}^{\mathrm{opt}}\right) + \Pr\left(\tilde{\mathbf{s}}(t) \in \mathcal{S}^{\mathrm{opt}} \,\middle|\, \tilde{\mathbf{s}}(t-D) \in \mathcal{S}\backslash\mathcal{S}^{\mathrm{opt}}\right) \Pr\left(\tilde{\mathbf{s}}(t-D) \in \mathcal{S}\backslash\mathcal{S}^{\mathrm{opt}}\right) \\ &= \omega_{t-D} + \Pr\left(\tilde{\mathbf{s}}(t) \in \mathcal{S}^{\mathrm{opt}} \,\middle|\, \tilde{\mathbf{s}}(t-D) \in \mathcal{S}\backslash\mathcal{S}^{\mathrm{opt}}\right) \Pr\left(\tilde{\mathbf{s}}(t-D) \in \mathcal{S}\backslash\mathcal{S}^{\mathrm{opt}}\right)\end{aligned} \tag{36}$$

Proof: Based on $\{\mathbf{s}(t)\}_{t=0}^{+\infty}$, we construct an auxiliary chain $\{\tilde{\mathbf{s}}(t)\}_{t=0}^{+\infty}$ by letting the global optimum of the original chain $\{\mathbf{s}(t)\}_{t=0}^{+\infty}$ to be an absorbing state. That is, whenever a state in $\mathcal{S}^{\mathrm{opt}}$ is reached, the system state will stay there. Moreover, $\tilde{\mathbf{s}}(t)$ is obviously a time-homogeneous Markov chain. Therefore, $\forall t > D$, Eq. (36) holds.

In the following, we analyze the worst case in terms of the convergence speed (only one BS updates its strategy in each iteration). In the proposed algorithm, the transition probability is given by $\frac{\exp\left\{\beta \hat{U}_k(t)\right\}}{\exp\{\beta U_k(t)\}+\exp\left\{\beta \hat{U}_k(t)\right\}}$. Because the state transition is bidirectional (i.e., time-reversible), if state i transfers to state j, state j can also transfer to state i, and the reverse transition probability is $\frac{\exp\{\beta U_k(t)\}}{\exp\{\beta U_k(t)\}+\exp\left\{\beta \hat{U}_k(t)\right\}}$. Thus, the lower bound of transition probability is $\xi = \min\limits_{\beta}\left(\frac{1}{1+\exp\left\{\beta\left|\hat{U}_k(t)-U_k(t)\right|\right\}}\right)$. Therefore,

$$\Pr\left(\tilde{\mathbf{s}}(t) \in \mathcal{S}^{\mathrm{opt}} \left| \tilde{\mathbf{s}}(t-D) \in \mathcal{S}\backslash\mathcal{S}^{\mathrm{opt}}\right.\right) \geq \xi^D. \tag{37}$$

Then, Eq. (36) results in

$$\omega_t \geq \omega_{t-D} + (1-\omega_{t-D})\,\xi^D. \tag{38}$$

Thus,

$$1-\omega_t \leq \left(1-\xi^D\right)(1-\omega_{t-D}). \tag{39}$$

Furthermore, we know $\omega_t \leq \omega_{t-1}$, $\forall t > 1$, since the chain $\{\tilde{\mathbf{s}}(t)\}_{t=0}^{+\infty}$ is absorbing. Through recursive iteration, we achieve

$$1-\omega_t \leq \left(1-\xi^D\right)^r(1-\omega_0), \quad r = \lfloor t/D \rfloor, \tag{40}$$

where $\lfloor t/D \rfloor$ denotes the largest integer not beyond t/D. Let τ be the random variable denoting the time when a optimal state in $\mathcal{S}^{\mathrm{opt}}$ is reached for the first time. Thus, $\tau = t$ represents the event $\{\tilde{\mathbf{s}}(t) \in \mathcal{S}^{\mathrm{opt}}\} \bigcap\limits_{n=0}^{t-1} \{\tilde{\mathbf{s}}(n) \in \mathcal{S}\backslash\mathcal{S}^{\mathrm{opt}}\}$, and $E[\tau]$ denotes the expected time to reach the global optimum. According to the property of absorbing Markov chains, $\Pr(\tau = t) = \omega_t - \omega_{t-1}, \forall t \geq 1$ with $\Pr(\tau = 0) = \omega_0$, so that $\{\omega_t\}$ leads to the distribution of τ. Then, applying the relation between the expected value of a non-negative random variable and its probability distribution, we have

$$E[\tau] = \sum_{t=0}^{\infty} \Pr(\tau > t) = \sum_{t=0}^{\infty}(1-\omega_t). \tag{41}$$

Now, based on (40) and (41), we can derive

$$\begin{aligned}E[\tau] &= \sum_{r=0}^{\infty}\sum_{m=0}^{D-1}(1-\omega_{rD+m}) \\ &\leq \sum_{r=0}^{\infty} D(1-\omega_0)\left(1-\xi^D\right)^r = \frac{D(1-\omega_0)}{\xi^D}.\end{aligned} \tag{42}$$

Besides, owing to the relationship between the two chains $\{\mathbf{s}(t)\}_{t=0}^{+\infty}$ and $\{\tilde{\mathbf{s}}(t)\}_{t=0}^{+\infty}$, we know the convergence time of $\{\tilde{\mathbf{s}}(t)\}_{t=0}^{+\infty}$ is exactly the time that $\{\mathbf{s}(t)\}_{t=0}^{+\infty}$ reaches the global optimum for the first time. Therefore, the proof is completed. ■

Because $\xi < 1$, the upper bound of the number of iterations for reaching the global optimum $\frac{D(1-\omega_0)}{\xi^D}$ increases with the one-step transition diameter D. It is known from the definition of D that D is directly related to the amount of BSs K. When K is larger, D generally gets larger as well, which leads to relatively slower convergence of the algorithm. Besides, since ξ monotonously decreases with the increasing of β, we can conclude that the average time needed to reach the global optimum will be longer when β gets larger, and vice versa. Furthermore, besides the first time to reach the global optimum, we are also concerned about the average time of returning to the global optimum after leaving that state. As we have proved in Theorem 4, the irreducible and aperiodic Markov chain $\{\mathbf{s}(t)\}_{t=0}^{+\infty}$ has an unique stationary distribution $\pi(\mathbf{s}) = \frac{\exp\{\beta\Phi(\mathbf{s})\}}{\sum_{\hat{\mathbf{s}}\in\mathcal{S}}\exp\{\beta\Phi(\hat{\mathbf{s}})\}}$. With a standard Markov chain analysis, starting from any state $\mathbf{s}$, the average time for returning to $\mathbf{s}$ is $\eta(\mathbf{s}) = \frac{1}{\pi(\mathbf{s})}$, which is inversely proportional to the stationary distribution probability [52]. Therefore, when β increases, the average time for returning to the optimal solution decreases since $\pi(\mathbf{s}^{\mathrm{opt}})$ gets larger.

V. Simulation Results

In this section, we use extensive simulations to verify the proposed energy-efficient BS sleeping algorithm. Similar to [4], [20], a heterogeneous network topology composed of five macro BSs and five micro BSs in 5×5 km^2 is considered for our simulations. A snapshot of cell coverage is plotted in Fig. 5. In the simulation, the maximal transmission power for macro and micro BSs is set to be 43dBm and 30dBm [4], respectively. Based on the linear relationship between transmission and operational power consumptions [4], we could derive the maximal operational powers for macro and micro BSs as 865W and 38W, respectively.

For the traffic model, we assume that file transmission requests at location $x \in \mathcal{A}$ follow a Poisson point process with arrival rate $\lambda(x)$ and average file size $1/\mu(x) = 100$ kbyte. In modeling the propagation environment, we use the modified COST 231 path loss model with macro BS height $h = 32$m and micro BS height $h = 12.5$m [4]. Other parameter settings are based on the IEEE 802.16m evaluation methodology document[5] [53]. To guarantee the system reliability, the system load threshold for all BSs are set to be $\rho^{\mathrm{th}} = 0.6$ [18]. For the

[5] Notably, in order to present more comprehensive numerical analyses, we slightly change one of these parameters during the simulation processes. Whenever we make the changes, we will explicitly refer to it.

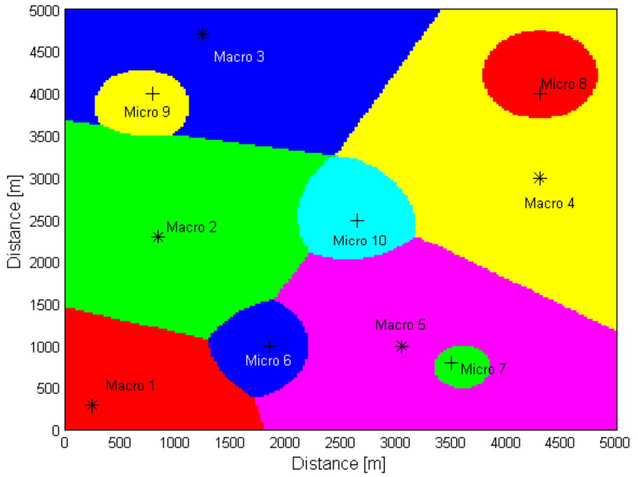

Fig. 5. Snapshot of cell coverage when all the BSs are on (BSs 1-5 are macro BSs, and BSs 6-10 are micro BSs, $q = 0.5$, $\rho^{\text{th}} = 0.6$, $\lambda(x) = 2 \times 10^{-6}$).

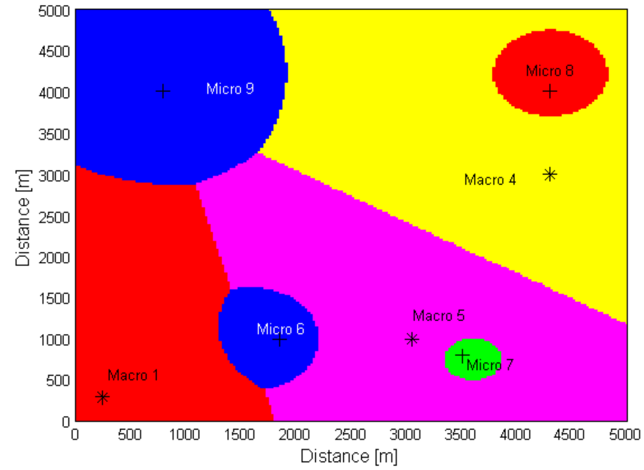

Fig. 6. Snapshot of cell coverage by the proposed BSs sleeping algorithm ($q = 0.5$, $\rho^{\text{th}} = 0.6$, $\lambda(x) = 2 \times 10^{-6}$).

cost function of energy consumption, we vary the portion of fixed power consumption q_i between 0 and 1 so that we can cover several types of BSs from energy-proportional BSs to non-energy-proportional BSs.

Algorithm 2: Greedy BS Switching-off Algorithm

Initialize $\mathcal{B}_{\text{on}} = \mathcal{K}$;
while $\mathcal{B}_{\text{on}} \neq \emptyset$
  for $k \in \mathcal{K}$
    while $\rho_i + \rho_{k \to i} \leq \rho_i^{\text{th}}, \forall i \in \mathcal{C}_k \cap \mathcal{B}_{\text{on}}$
      Calculate $\Delta(k) = E(\mathcal{B}_{\text{on}}) - E(\mathcal{B}_{\text{on}} \setminus \{k\})$;
    end while
  end for
  if No BS can be switched off, **then** stop the algorithm;
  else, find BS $k^* = \arg\max_{k \in \mathcal{B}_{\text{on}}} \Delta(k)$, $\mathcal{B}_{\text{on}} \leftarrow \mathcal{B}_{\text{on}} - \{k\}$;
end while

Algorithm 3: SWES Algorithm

Initialize $\mathcal{B}_{\text{on}} = \mathcal{K}$;
while $\mathcal{B}_{\text{on}} \neq \emptyset$
  for $k \in \mathcal{K}$
    while $\rho_i + \rho_{k \to i} \leq \rho_i^{\text{th}}, \forall i \in \mathbf{N}_k$. Here, $\mathbf{N}_k = \mathcal{C}_k \cap \mathcal{B}_{\text{on}}$.
      Calculate $\Delta(k) = \frac{1}{|\mathbf{N}_k|} \sum_{i \in \mathbf{N}_k} (\rho_i + \rho_{k \to i})$;
    end while
  end for
  if No BS can be switched off, **then** stop the algorithm;
  else, find BS $k^* = \arg\min_{k \in \mathcal{B}_{\text{on}}} \Delta(k)$, $\mathcal{B}_{\text{on}} \leftarrow \mathcal{B}_{\text{on}} - \{k\}$;
end while

A. Convergence of the Proposed Algorithm

1) Static Networking Scenarios: The convergence behavior of the proposed algorithm is shown in Fig. 7, and the convergence curves of the greedy algorithm in [4], [16] and the SWES (switching-on/off based energy saving) algorithm in [18] are presented for comparison. In order to prove the optimality of our proposed algorithm, the globally optimal solution is plotted by exhaustive search. The energy saving ratio is used for the performance metric, which is defined as: 1-[the ratio between energy consumptions when the maximum number of BSs are turned off and when all BSs are turned on]. These results are obtained under homogeneous traffic distribution, i.e., $\lambda(x) = \lambda$ for all $x \in \mathcal{A}$. As depicted in Fig. 7, the network utilities by the greedy algorithm and the proposed algorithm are updated at each iteration and both greatly improved at the convergence time. Furthermore, our proposed algorithm can finally achieve the global optimum, while the greedy algorithm only obtains a local optimum. In contrast, the performance of the SWES algorithm is much worse, since it is a decentralized algorithm which uses the network-impact for decision metric and requires only local information. It should be noted that the greedy algorithm and the SWES algorithm converge faster than our proposed algorithm, since they use a deterministic updating method and might converge to a local optimum. Instead, our proposed algorithm employs a probabilistic updating method which could get rid of the local optimum at the expense of more time on the solution exploration. Moreover, please refer to Table II for more comprehensive comparisons on the three algorithms.

We now analyze in more details the convergence behavior of the proposed algorithm. Fig. 8 plots the evolution of the system load of each BS versus the number of iterations. The system load of each BS is updated along with the updating of BS sleeping strategy at each iteration, and the convergence is achieved in about 50 iterations. We can observe that the system load of macro BSs 2,3 and micro BS 10 converges to 0, which means they are turned off to save the power consumption while guaranteeing the system load of other BSs not beyond the threshold ρ^{th}. When macro BSs 2, 3 and micro BS 10 are switched off, those UEs originally associated with the switched-off BSs need to be transferred to the neighboring

TABLE II
ALGORITHMS COMPARISON.

	Performance of solution	Operation manner	Required information	Complexity of strategy updating	Convergence speed
Proposed algorithm	Global optimum	Decentralized	Local information	$O(1)$	Relatively slow
Greedy algorithm [4], [16]	Local optimum	Centralized	Global information	$O(K^2)$	Fast
SWES algorithm [18]	Feasible and dynamic solution	Decentralized	Local information	$O(1)$	Medium

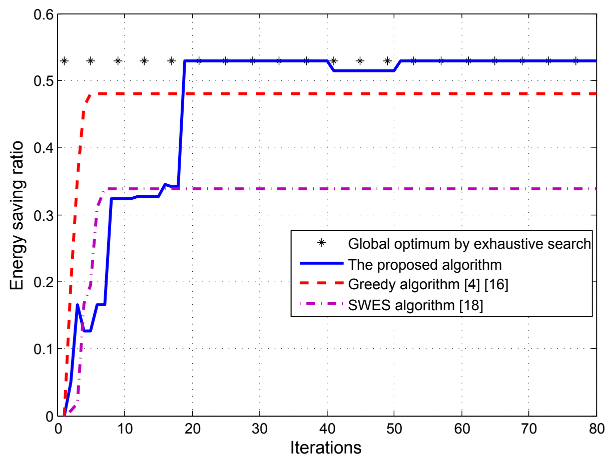

Fig. 7. Convergence and optimality ($q = 0.5$, $\rho^{\text{th}} = 0.6$, $\lambda(x) = 2 \times 10^{-6}$).

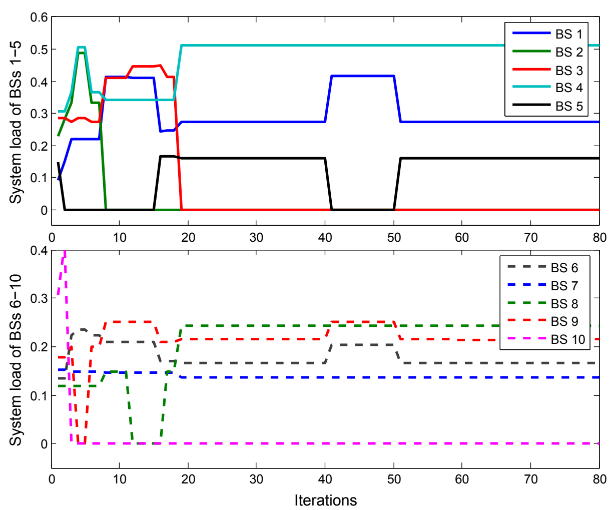

Fig. 8. The evolution of each BS's system load versus the iterations ($q = 0.5$, $\rho^{\text{th}} = 0.6$, $\lambda(x) = 2 \times 10^{-6}$).

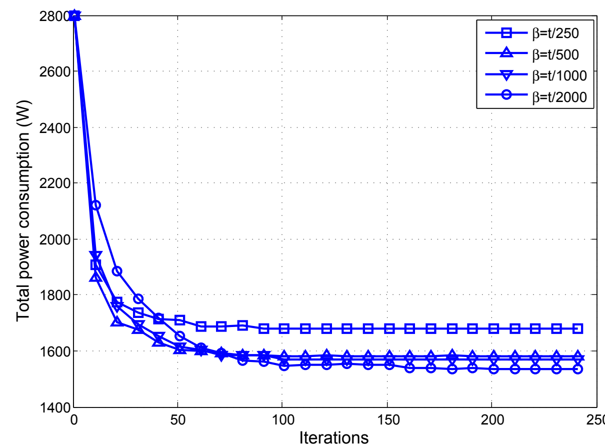

Fig. 9. Comparison of convergence speed of the proposed algorithm when changing β ($q = 0.5$, $\rho^{\text{th}} = 0.6$, $\lambda(x) = 2 \times 10^{-6}$).

BSs. To get a clear and vivid understanding of the users transferring, we plot the snapshots of cell coverage before and after macro BSs 2, 3 and micro BS 10 are switched off in Fig. 5 and Fig. 6, respectively. By comparing the two figures, we can see that the users originally served by micro BS 10 are transferred to macro BSs 4 and 5. When macro BSs 2 and 3 are switched off, micro BS 9 located near the boundary between BS 2 and BS 3 extends its coverage to serve a larger area, and the left users are transferred to macro BSs 1, 4 and 5. Overall, besides the micro BSs nearby, macro BSs take the primary responsibility of compensating the coverage.

According to Theorems 5 and 6, the tradeoff between optimality and convergence speed is controlled by the learning parameter β in Step 3 of the proposed algorithm, which balances the tradeoff between exploration and exploitation [45]. Note that the smoothing factor β here is analogous to the inverse of temperature in simulated annealing. A small β represents extensive space search with slow convergence while a large β represents limited space search with fast convergence [37], [46]. Therefore, it is advisable that at the beginning phase the value of β is set to be a small number for extensive space search, and keeps increasing as the proposed algorithm iterates for accelerating the convergence [46], [47], [50].

In practice, to achieve a tradeoff between optimality and convergence speed, engineering approaches such as $\beta = \frac{t}{c}$, where t is the iteration step and c is a constant coefficient, can be utilized, as in [47], [50]. To investigate the impact of coefficient c on the convergence, we plot the convergence curves as for different selections of c in Fig. 9. The simulation results are obtained by taking the average value over 1000 trials. We can find that when c is larger, the achieved solution is of less power consumption, while the convergence needs more time. In contrast, when c takes smaller value like 250, the proposed algorithm can converge fast in 50 iterations, but the obtained solution consumes higher power. It is because larger c leads to more extensive solution space search which can easily find the globally optimal solution, however, the extensive search also results in slower convergence. According to Fig. 9, it is better to select c as 1000, since it allows the algorithm to convergence within 100 iterations and also achieve a near-optimal solution.

2) Dynamic Networking Scenarios: We further study the convergence performance of the proposed algorithm in dy-

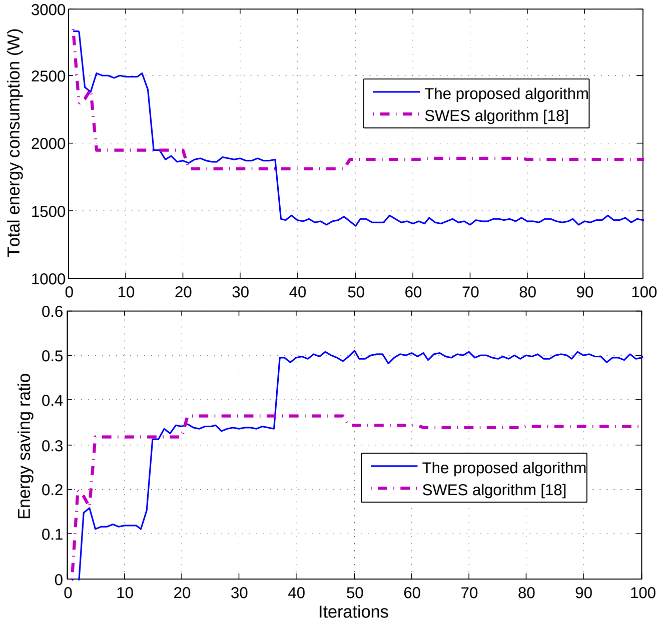

Fig. 10. Convergence performance in the channel dynamic scenario ($q = 0.5$, $\rho^{\text{th}} = 0.6$, $\lambda(x) = 2 \times 10^{-6}$).

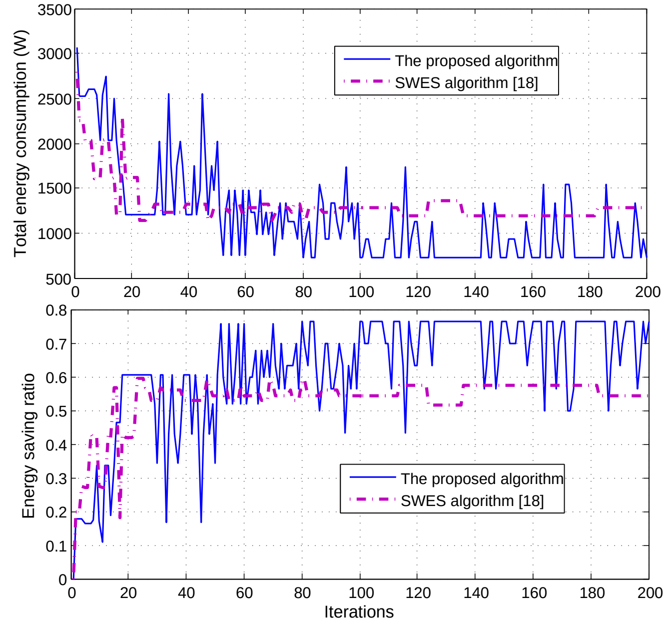

Fig. 11. Convergence performance in the traffic dynamic scenario ($q = 0.5$, $\rho^{\text{th}} = 0.6$).

namic scenarios. Firstly, we consider the channel dynamic scenario where channels are time-varying during the convergence of the algorithm. The channels are assumed to undergo Rayleigh fading with unit mean. Moreover, we consider fast fading channels that vary at different iteration slots. Secondly, we take account of the traffic dynamic scenario where the evolution of the file size follows the long range dependence (LRD) distribution [54]–[56]. The LRD traffic is generated by using the approach in [56] with the same parameter setting therein. Due to dynamic changing of channel and traffic, obviously the optimal solution should be a dynamic solution. However, since the environment (channel, traffic) dynamically and stochastically varies, the channel and traffic characteristics in the future time slots are unknown a priori, and thus the dynamic optimal solution to the BS sleeping problem cannot be achieved a priori.

Fig. 10 and Fig. 11 plots the convergence behavior of the proposed algorithm under channel dynamics and traffic dynamics, respectively. Since only the SWES algorithm [18] is originally designed for dynamic systems, we merely plot its dynamic solution for performance comparison. It is seen in Fig. 10 that both algorithms can well adapt to the channel dynamics. After 50 iterations, both algorithms can greatly decrease the total energy consumption and improve the energy saving ratio. Moreover, the proposed algorithm can achieve better performance than the SWES algorithm. Besides, by comparing Fig. 10 and Fig. 11, we can see that the traffic dynamics have a greater impact than the channel dynamics on the algorithms. More sharp vibrations can be observed in Fig. 11. It is because the LRD traffic tends to arrive in clusters, causing the burst phenomenon [56]. However, Fig. 11 shows the proposed algorithm can effectively decrease the total energy consumption and improve the energy saving ratio albeit experiencing sharp vibrations. In contrast, the SWES algorithm can well adapt to the LRD traffic dynamics, but obtains a relatively worse solution.

In practical implementations, the BS switching algorithm cannot keep running to adapt to the dynamic environment. Once some BSs are switched off, the UEs within their coverage will need to be transferred to other BSs, thus incurring a large amount of handover. If the algorithm runs frequently, the handover cost becomes a major concern. Therefore, we only re-start the algorithm when the traffic profile experiences significant variations. From many measurement data in real networks [9], [17], [18], the traffic pattern clearly varies over time, but could still be assumed almost constant during a certain period of time, e.g., one hour [4]. Therefore, the timescale of running the BS switching algorithm is at the scale of an hour similar to [4], and the implementation in each period is in a quasi-static manner based on the average channel gain and expected traffic arrival.

B. Energy Saving Performance

Hereinafter, we compare the performance of the proposed algorithm with those of the existing algorithms under various parameter configurations in quasi-static environment.

Fig. 12 plots the energy saving ratio by different algorithms versus the arrival rate $\lambda(x)$. As we have theoretically proved, our algorithm can converge to the global optimum. In contrast, the greedy algorithm [4], [16] finds a locally optimal solution[6], while the SWES algorithm [18] provides the worst energy-saving performance because it mainly focuses on the dynamic environment with less information exchange. Moreover, it can be observed that the energy saving ratio decreases with the

[6]It should be noted that the greedy algorithm can sometimes find the global optimum. The reasons may be 1) the greedy search route could lead to global optimum in some particular cases, 2) there is only one local/global optimum.

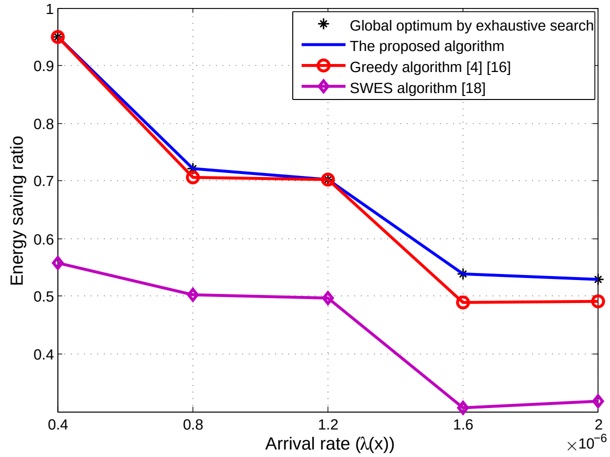

Fig. 12. The energy saving ratio by different algorithms versus the arrival rate ($q = 0.5$, $\rho^{\text{th}} = 0.6$).

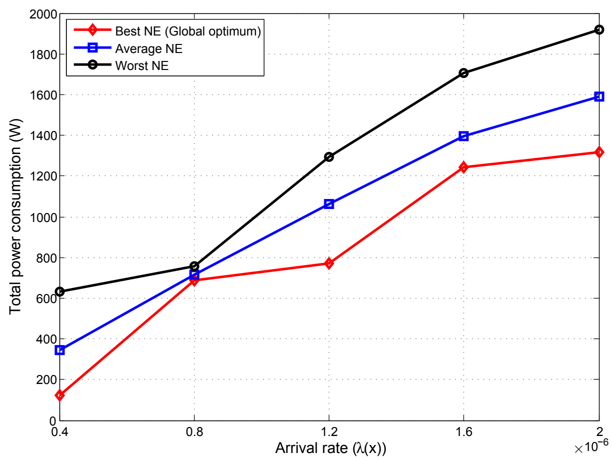

Fig. 13. The power consumption by different solutions versus the arrival rate ($q = 0.5$, $\rho^{\text{th}} = 0.6$).

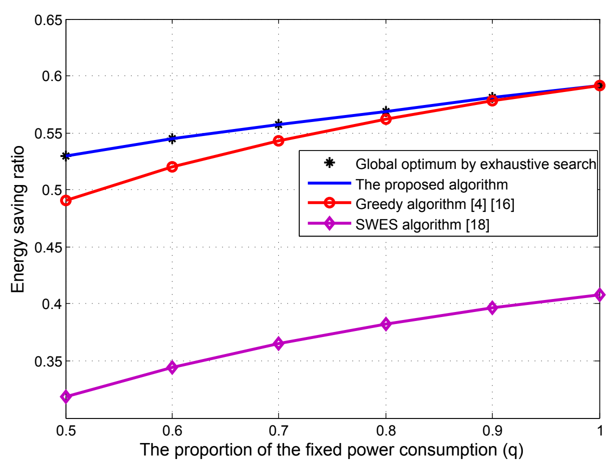

Fig. 14. The energy saving ratio by different algorithms versus the proportion of the fixed power consumption ($\rho^{\text{th}} = 0.6$, $\lambda(x) = 2 \times 10^{-6}$).

increasing of the arrival rate. The reason is that when the arrival rate becomes larger, the traffic load will get heavier given the fixed file size $\mu(x)$. In order to guarantee the coverage, less BSs can be switched off. Therefore, the energy

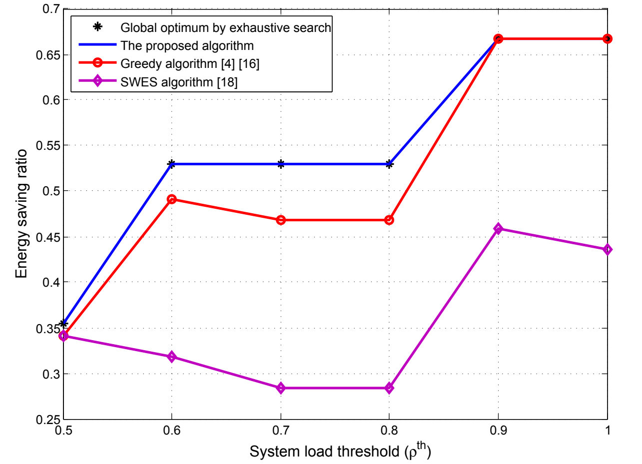

Fig. 15. The energy saving ratio by different algorithms versus the system load threshold ($q = 0.5$, $\lambda(x) = 2 \times 10^{-6}$).

saving ratio decreases relatively. Besides, it is observed that the curves are not smooth. The reason is as follows. When the arrival rate $\lambda(x)$ decreases from 1.2×10^{-6} to 0.8×10^{-6}, only a micro BS could be switched off, which results in marginal variation of the energy saving ratio. However, when the arrival rate $\lambda(x)$ decreases from 0.8×10^{-6} to 0.4×10^{-6}, a macro BS could be switched off, which achieves significant variation of the energy saving ratio because switching off a macro BS can save more energy than switching off a micro BS. Also, for the sake of comparison, Fig. 13 plots the power consumption by different solutions versus the arrival rate. It is seen that the best NE consumes the least power same to the global optimum, as theoretically proved in Theorem 3. Besides, the performance of the average NE follows, and the worst NE consumes the most power.

In Fig. 14, we plot the energy saving ratio achieved by different algorithms versus the proportion of the fixed power consumption q. It can be clearly seen that larger energy saving can be expected from larger values of q. In other words, the farther the BSs are from energy-proportional operation mode, the larger energy conservation can be expected. This is because the energy-saving benefit from turning off one BS mainly comes from reducing the fixed power consumption term [4]. As for the performance comparison, we can notice that our proposed algorithm achieves the maximum energy saving, the greedy algorithm inferior, and the SWES algorithm worst. Furthermore, when the proportion of the fixed power consumption q increases, the gap between the greedy solution and the global optimum becomes smaller, and the greedy algorithm can obtain a near-optimal solution.

Besides, we present the energy saving ratio by different algorithms versus the threshold of the system load ρ^{th} in Fig. 15. Also, the proposed algorithm achieves the best performance, while the SWES algorithm possesses the worst solution. Moreover, as shown in Fig. 15, more energy saving can be achieved when the system load threshold gets larger, which matches our common sense because larger threshold means larger feasible solution space. Additionally, it can be observed in Fig. 15 that there are some distinct falling points

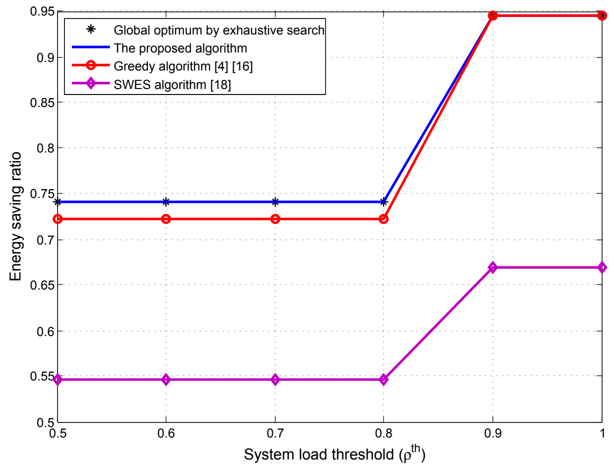

Fig. 16. The energy saving ratio by different algorithms versus the system load threshold in an inhomogeneous case. As an example of inhomogeneous traffic loads [4], a linearly increasing load along the diagonal direction from right bottom (10^{-7}) to left top (10^{-6}) is considered. Besides, the proportion of the fixed power consumption for micro BSs is set to be $q = 0.4$, and that for macro BSs is set to be $q = 0.6$.

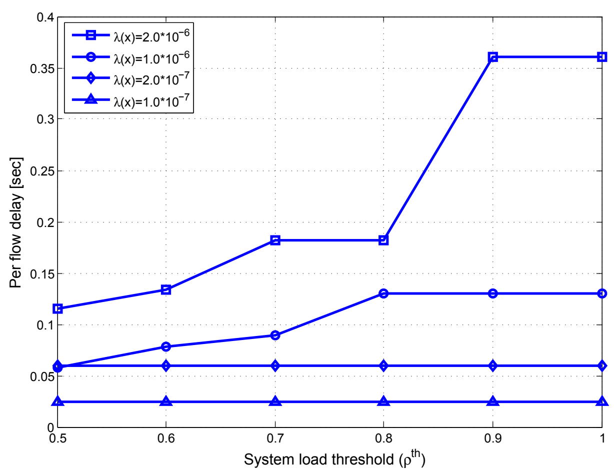

Fig. 17. The per flow delay achieved by the proposed algorithm versus the system load threshold ρ^{th} under different arrival rates ($q = 0.5$).

for the greedy algorithm and SWES algorithm. It is because both of the two algorithms search better solution in a greedy manner. Therefore, the obtained solution may not match the general trend. Similar performance behaviors can be found in [4]. In addition, Fig. 16 shows the energy saving ratio by different algorithms versus the system load threshold ρ^{th} in an inhomogeneous case, which also validates the performance of the proposed algorithm.

On the other hand, increasing the system load threshold would inevitably bring longer service delay, as analyzed in Section II. In order to evaluate the impact of the selection of ρ^{th} on the service delay, we depict the performances of per flow delay achieved by the proposed algorithm versus the system load threshold under different arrival rates in Fig. 17. It is clearly demonstrated that increasing the system load threshold results in more severe delay when the arrival rate is large, and does not cause significant impact when the arrival rate is small. Furthermore, given the same system load threshold ρ^{th}, larger

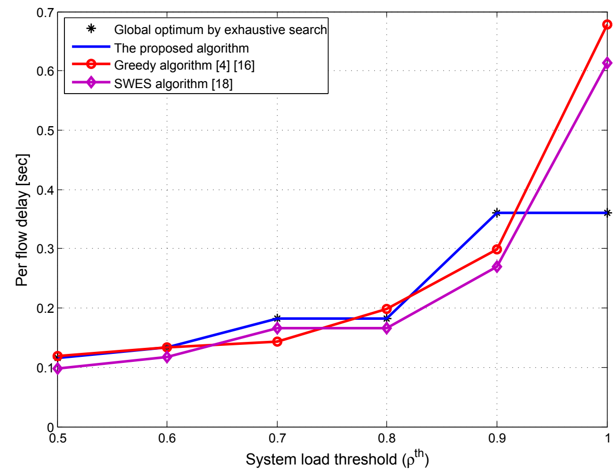

Fig. 18. Delay comparison of different algorithms ($q = 0.5$, $\lambda(x) = 2 \times 10^{-6}$).

arrival rate corresponds to more severe delay. In addition, we compare the per flow delay of the proposed algorithm and the state-of-the-art methods in Fig. 18. According to [4], [20], the delay is calculated by $\psi = \sum_{i \in \mathcal{B}_{\text{on}}} \frac{\rho_i}{1-\rho_i}$, which is only determined by the traffic load ρ_i. Besides, the traffic load ρ_i is subject to Eq. (6b) as a constraint in our BS sleeping problem. Since ρ_i is strictly guaranteed to be less than a threshold ρ_i^{th}, the delay performance by different algorithms can be all guaranteed, as shown in Fig. 18. However, there is no obvious superiority or inferiority for each algorithm in terms of the delay performance. The reason is that the delay performance (determined by ρ_i) is formulated as a constraint while minimizing the total energy expenditure is formulated as the optimization goal, as given by Eq. (6).

VI. Conclusion

In this paper, we have proposed a distributed cooperative framework to improve the energy efficiency of the green cellular networks. Within this framework, the neighboring BSs cooperated to optimize the switching strategies in order to maximize the energy saving while guaranteeing users' minimal service requirements. The inter-BS cooperation was formulated based on the principle of ecological self-organization. Specifically, an interaction graph was first defined to capture the network impact of a BS switching operation. Then, we formulated the problem of energy saving as a constrained graphical game with local cooperation, where each BS acted as a game player under the constraint of traffic load. The constrained graphical game was proved to be an exact constrained potential game. Furthermore, we proved the existence of a generalized Nash equilibrium (GNE), and the best GNE coincided with the optimal solution of total energy consumption minimization. Accordingly, we designed a decentralized iterative algorithm to find the globally optimal solution, where only local information exchange among the neighboring BSs was needed. Theoretical analysis and simulation results illustrated effectiveness and efficiency of the proposed algorithm.

Future work may jointly consider the optimal BS switching and user association scheme, since this paper only studies a simple signal strength based users association. Secondly, it would be interesting and challenging to extend the model to a more practical case where various BSs belong to different service provider and some BSs may not be willing to participate in the altruistic cooperation. Besides, the switching off transients of the BSs will also be studied in the future work.

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REFERENCES

[1] M. A. Marsan and M. Meo, "Network sharing and its energy benefits: A study of European mobile network operators," in *Proc. IEEE GLOBECOM*, 2013.

[2] J. Wu, S. Rangan, and H. Zhang, Eds., *Green Communications: Theoretical Fundamentals, Algorithms and Applications*, 1st ed. CRC Press, Sep. 2012.

[3] H. Zhang, A. Gladisch, M. Pickavet, Z. Tao, and W. Mohr, "Energy efficiency in communications," *IEEE Commun. Mag.*, vol. 48, no. 11, pp. 48-49, Nov. 2010.

[4] K. Son, H. Kim, Y. Yi, and B. Krishnamachari, "Base station operation and user association mechanisms for energy-delay tradeoffs in green cellular networks," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1525-1536, Sep. 2011.

[5] Z. Niu, "TANGO: Traffic-aware network planning and green operation," *IEEE Wireless Commun.*, vol. 18, no. 5, pp. 25-29, Oct. 2011.

[6] W. Wong, Y. Yu, and A. Pang, "Decentralized energy-efficient base station operation for green cellular networks," in *Proc. IEEE GLOBECOM*, 2012.

[7] Z. Hasan, H. Boostanimehr, and V. K. Bhargava, "Green cellular networks: a survey, some research issues and challenges," *IEEE Commun. Surveys & Tutorials*, vol. 13, pp. 524-540, 2011.

[8] E. Oh, B. Krishnamachari, X. Liu, and Z. Niu, "Toward dynamic energy efficient operation of cellular network infrastructure," *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 56-61, Jun. 2011.

[9] M. A. Marsan, L. Chiaraviglio, D. Ciullo, and M. Meo, "Optimal energy savings in cellular access networks," in *Proc. first International Workshop on Green Communications*, pp. 1-5, June 2009.

[10] L. Chiaraviglio, D. Ciullo, M. Meo, and M. A. Marsan, "Energy-aware UMTS access networks," in *Proc. WPMC*, pp. 1-5, Sep. 2008.

[11] M. A. Marsan and M. Meo, "Energy efficient management of two cellular access networks," in *Proc. ACM SIGMETRICS*, pp. 69-73, Mar. 2010.

[12] R. M. Karp, "Reducibility among combinatorial problems," in *Complexity of Computer Computations.* Plenum Press, pp. 85-103, 1972.

[13] M. R. Garey and D. S. Johnson, *Computers and Intractability; A Guide to the Theory of NP-Completeness.* W. H. Freeman & Co., 1990.

[14] F. Han, Z. Safar, and K. J. Ray Liu, "Energy-efficient base-station cooperative operation with guaranteed QoS," *IEEE Trans. Commun.*, vol. 61, no. 8, pp. 2126-2136, Aug. 2013.

[15] Y. S. Soh, T. Q. S. Quek, M. Kountouris, and H. Shin, "Energy efficient heterogeneous cellular networks," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 5, pp. 840-850, May 2013.

[16] S. Kim, S. Choi, and B. G. Lee, "A joint algorithm for base station operation and user association in heterogeneous networks," *IEEE Commun. Lett.*, vol. 17, no. 8, pp. 1552-1555, Aug. 2013.

[17] S. Zhou, J. Gong, Z. Yang, Z. Niu, and P. Yang, "Green mobile access network with dynamic base station energy saving," in *Proc. ACM International Conference on Mobile Computing and Networking*, 2009.

[18] E. Oh, K. Son, and B. Krishnamachari, "Dynamic base station switching-on/off strategies for green cellular networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2126-2136, May 2013.

[19] W. Guo and T. O'Farrell, "Dynamic cell expansion with self-organizing cooperation," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 5, pp. 851-860, May 2013.

[20] R. Li, Z. Zhao, X. Chen, J. Palicot, and H. Zhang, "TACT: A transfer actor-critic learning framework for energy saving in cellular radio access networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 4, pp. 2000-2011, Apr. 2014.

[21] T. Han and N. Ansari, "On greening cellular networks via multicell cooperation," *IEEE Wireless Commun.*, pp. 82-89, Feb. 2013.

[22] 3GPP, *Feasibility Study for Evolved UTRA and UTRAN*, TR 25.912v10.0.0, Apr. 2011.

[23] 3GPP, *Requirements for Further Advancements for E-UTRAN*, TR 36.913v10.0.0, Apr. 2011.

[24] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, "Modeling and analysis of K-tier downlink heterogeneous cellular networks," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 550-560, Apr. 2012.

[25] H. Kim, G. Veciana, X. Yang, and M. Venkatachalam , "Distributed α-optimal user association and cell load balancing in wireless networks," *IEEE/ACM Trans. Netw.*, vol. 20, no. 1, pp. 177-190, Feb. 2012.

[26] K. Son, S. Chong, and G. Veciana, "Dynamic association for load balancing and interference avoidance in multi-cell networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3566-3576, Jul. 2009.

[27] M. F. Hossain, K. S. Munasinghe, and A. Jamalipour, "Distributed inter-BS cooperation aided energy efficient load balancing for cellular networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 11, pp. 5929-5939, Nov. 2013.

[28] S. Camazine, J. L. Deneubourg, N. R. Franks, J. Sneyd, G. Theraulaz, and E. Bonabeau, *Self-Organization in Biological Systems*, Princeton University Press, 2002.

[29] C. Prehofer and C. Bettstetter, "Self-organization in communication networks: principles and design paradigms," *IEEE Commun. Mag.*, vol. 43, no. 7, 78-85, July 2005.

[30] G. Debreu, "A social equilibrium existence theorem," in *Proc. National Acad. Sci. United States of Amer.*, vol. 38, no. 10, 886-893, Oct. 1952.

[31] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave n-person games," *Econometrica*, vol. 33, no. 3, pp. 520-534, 1965.

[32] S. M. Perlaza, H. Tembine, S. Lasaulce, and M. Debbah, "Quality-of-service provisioning in decentralized networks: a satisfaction equilibrium approach," *IEEE J. Sel. Topics Signal Process.*, vol. 6, no. 2, pp. 104-116, Apr. 2012.

[33] J.-S. Pang, G. Scutari, F. Facchinei, and C. Wang, "Distributed power allocation with rate constraints in Gaussian parallel interference channels," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3471-3489, Aug. 2008.

[34] H. Zhang, L. Venturino, N. Prasad, P. Li, S. Rangarajan, and X. Wang, "Weighted sum-rate maximization in multi-cell networks via coordinated scheduling and discrete power control," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 6, pp. 1214-1224, Jun. 2011.

[35] A. Bousia, E. Kartsakli, A. Antonopoulos, L. Alonso, and C. Verikoukis, "Game theoretic approach for switching off base stations in multi-operator environments," in *Proc. IEEE ICC*, 2013.

[36] J. Zheng, Y. Cai, Y. Xu, and A. Anpalagan, "Distributed channel selection for interference mitigation in dynamic environment: A game-theoretic stochastic learning solution," *IEEE Trans. Veh. Tech.*, vol. 63, no. 9, pp. 4757-4762, Nov. 2014.

[37] J. Zheng, Y. Cai, Y. Liu, Y. Xu, B. Duan, and X. Shen, "Optimal power allocation and user scheduling in multicell networks: Base station cooperation using a game-theoretic approach," *IEEE Trans. Wireless Commun.*, vol. 13, no. 12, pp. 6928-6942, Dec. 2014.

[38] J. Zheng, Y. Cai, N. Lu, Y. Xu, and X. Shen, "Stochastic game-theoretic spectrum access in distributed and dynamic environment," *IEEE Trans. Veh. Tech.*, to appear, 2015. DOI: 10.1109/TVT.2014.2366559.

[39] J. Zheng, Y. Cai, and A. Anpalagan, "A stochastic game-theoretic approach for interference mitigation in small cell networks," *IEEE Commun. Lett.*, vol. 19, no. 2, pp. 251-254, Feb. 2015.

[40] J. Zheng, Y. Cai, and A. Anpalagan, "A game-theoretic approach to exploit partially overlapping channels in dynamic and distributed networks," *IEEE Commun. Lett.*, vol. 18, no. 12, pp. 2201-2204, Dec. 2014.

[41] J. Zheng, Y. Cai, W. Yang, Y. Wei, and W. Yang, "A Fully Distributed Algorithm for Dynamic Channel Adaptation in Canonical Communication Networks," *IEEE Wireless Commun. Lett.*, vol. 2, no. 5, pp. 491-494, Oct. 2013.

[42] J. Zheng, Y. Cai, and D. Wu, "Subcarrier allocation based on correlated equilibrium in multi-cell OFDMA systems," *EURASIP J. Wireless Comm. and Netw.*, vol. 2012, pp. 233-245, 2012.

[43] Y. Cai, J. Zheng, Y. Wei, Y. Xu, and A. Anpalagan, "A joint game-theoretic interference coordination approach in uplink multi-cell OFDMA networks," *Wireless Pers. Commun.*, vol. 80, no. 3, pp. 1203-1215, Feb. 2015.

[44] D. Monderer and L. S. Shapley, "Potential games," *Games and Economic Behavior*, vol. 14, pp. 124-143, 1996.

[45] H. P. Young, *Individual Strategy and Social Structure.* Princeton Univ. Press, 1998.

[46] Y. Song, C. Zhang, and Y. Fang, “Joint channel and power allocation in wireless mesh networks: a game theoretical perspective,” *IEEE J. Sel. Areas Commun.*, vol. 26, no. 7, pp. 1149-1159, Sep. 2008.

[47] Y. Song, S. H. Y. Wong, and K.-W. Lee, “Optimal gateway selection in multi-domain wireless networks: a potential game perspective,” in *Proc. ACM MOBICOM*, pp. 325-336, 2011.

[48] P. J. M. van Laarhoven and E. H. L. Aarts, *Simulated Annealing: Theory and Applications.* Holland: Reidel, 1987.

[49] R. S. Sutton and A. G. Barto, *Reinforcement Learning: An Introduction.* Cambridge, MA: MIT Press, 1998.

[50] G. Arslan, J. Marden, and J. Shamma, “Autonomous vehicle-target assignment: A game theoretical formulation,” *ASME Journal of Dynamic Systems, Measurement and Control*, pp. 584-596, 2007.

[51] J. Marden, G. Arslan, and J. Shamma, “Cooperative control and potential games,” *IEEE Trans. Syst., Man, Cybern.*, vol. 39, no. 6, pp. 1393-1407, 2009.

[52] C. M. Grinstead and J. Snell, *Introduction to Probability.* American Mathematical Society, 1997.

[53] IEEE 802.16 Boradband Wireless Access Working Group, *IEEE 802.16m evaluation methodology document (EMD),* Jul. 2008. [Online]. Available: http://ieee802.org/16.

[54] H. Yousefi’zadeh and E. A. Jonckheere, “Dynamic neural-based buffer management for queuing systems with self-similar characteristics,” *IEEE Trans. Neural Netw.*, vol. 16, no. 5, pp. 1163-1173, Sep. 2005.

[55] H. Yousefi’zadeh, “A neural-based technique for estimating self-similar traffic average queueing delay,” *IEEE Commun. Lett.*, vol. 6, no. 10, pp. 419-421, Oct. 2002.

[56] R. H. Fares and M. E. Woodward, “The use of long range dependence for network congestion prediction,” in *Proc. the First International Conference on Evolving Internet*, pp. 119-124, Aug. 2009.